α-MATHEMATICS

Grade 10 **Time: 2 hours**

Examiner: Pieter van Onselen **Total: 130 marks**

Moderator: Lanice Liebenberg

INSTRUCTIONS and INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 7 pages and an answer sheet of 2 pages.
2. Answer ALL 7 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, must be given correct to two decimal places, where necessary.
6. Clearly show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 [20 marks]

This question must be answered on the **answer sheet**.

Every question has **ONLY** one correct answer for **TWO** marks each. Mark the correct answer with an **X** on the answer sheet.

1.1 Simplify: $\left(\sqrt{-9}\right)$

(A) 3 **(B)** -3 **(C)** $3i$ **(D)** $\sqrt{3}i$

1.2 60^0 is equivalent … radians

(A) $\frac{\pi}{2}$ radians **(B)** $1{,}047$ radians **(C)** $\frac{1}{3\pi}$ radians **(D)** $\frac{1}{3}$ radians

1.3 The conjugate of $2 - i$ is

(A) $-2 - i$ **(B)** $2 + i$ **(C)** i **(D)** $\boldsymbol{-2 + i}$

1.4 Which of the following statements are true:

(A) An identity matrix is sometimes a square matrix.

(B) The dimensions of a transposed matrix and the original matrix always remain the same.

(C) The determinant of an identity matrix is always 1.

(D) The determinant of a matrix's answer is also a matrix.

1.5 Given: $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$

Which operation of the following operations is possible?

(A) $A \times B$

(B) $A + B$

(C) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \times B + A$

(D) $B - A$

1.6 Given:

$$\begin{bmatrix} 2 & 3 \\ -1 & \boldsymbol{2a} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

If $x = 1$, determine the value of a.

(A) 1 **(B)** Undefined **(C)** 0 **(D)** 2

1.7 Given vectors $\boldsymbol{a} = (1; -2)$, $\boldsymbol{b} = (x; -3)$ and the dot product of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is -1. Determine the value of x.

(A) 7 **(B)** -7 **(C)** -5 **(D)** 5

1.8 The real part of $(1 + i)(2 - 3i)$ is ...

(A) 5 **(B)** 2 **(C)** -3 **(D)** -1

1.9 1 radian is equiivalent to

(A) $60°$ **(B)** $\frac{\pi}{180}$ **(C)** $\frac{90^0}{\pi}$ **(D)** $57{,}3°$

1.10 If $\frac{f(x)}{x^3-2x^2}$ is decomposed into partial fractions, then $\frac{f(x)}{x^3-2x^2} \equiv$ **...**

(A) $\frac{A}{x} + \frac{B}{x} + \frac{C}{x-2}$

(B) $\frac{A}{x^2} + \frac{B}{x-2}$

(C) $\frac{\boldsymbol{Ax+B}}{x^2} + \frac{B}{x-2}$

(D) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

Question 2 [25 marks]

2.1 Determine the following:

(a) $x^2 + 4 = 0$ (2)

(b) $\frac{-2}{i}$ (3)

2.2 Determine the value of the following and express the answer in the form $a + bi$ where a and b represent real numbers.

(a) $(2i - 3)^*$ (* is the conjugate of the complex number) (1)

(b) $i^{206}(2 - 3i)^2$ (5)

(c) $\frac{2+i}{2+2i}$ (5)

2.3. Given $(2a - i) - (2 - 3ai)$

(a) Determine the value of a to make the expression purely imaginary. (3)

(b) Subsequently determine the imaginary number. (2)

2.4 Find the solution of $(-1 - 2i) + (5 + 2i)$ graphically. Use **DIAGRAM SHEET 1** on the answersheet. (4)

Question 3 [18 marks]

3.1 Given the following system of equations:

$$x + y - z = -5$$
$$-3x - 2y - z = 0$$
$$-3y + z = 1$$

(a) Write the system of equations in matrix form. (3)

(b) Use Cramer's method and calculate the value of z. (7)

3.2 Given the following system of equations:

$x - y = 5$ and $ax + y = 7$.

(a) Use Cramer's method and show that $x = \frac{12}{1+a}$. (5)

(b) If $x = 4$, determine the value of a. (3)

Question 4 [17 marks]

4.1 Given the matrix $P = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$: (5)

A	B	C	D	E	F	G
$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ -2 & -3 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$	$(1 \quad 4)$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	No solution

Use A-F to answer the following questions.

For example: (a) D (b) C etc.

(a) P^T

(b) Identity matrix I

(c) $(-1 \quad 1) \times P$

(d) Zero-matrix

(e) $-1 \times P$

4.2 Given the matrixes: A=$\begin{pmatrix} 1 & \frac{1}{2} \\ -2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ -2 & 0 \end{pmatrix}$.

Perform the following matrix operations. If it is not possible, explain why not.

(a) $detB$ (2)

(b) $B - 2A$ (2)

(c) $C \times B$ (3)

4.3 Given the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \\ 0 & 2 & -2 \end{pmatrix}$. Determine the determinant of A. (5)

Question 5 [16 marks]

5.1 Decompose $\frac{5x^2+6x+3}{x^3+2x^2+x}$ into partial fractions.

First factorise the denominator. (10)

5.2 $\frac{(1-x+x^3)}{x^4-x^3} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{1}{x-1}$ (6)

Given: $1 - x + x^3 \equiv Ax^2(x-1) + Bx(x-1) + C(x-1) + x^3$

Determine the value of A, B and C by decomposing into partial fractions.

Question 6 [14 marks]

6.1 Given $f(x) = x^2 - 2$ and $g(x) = \sqrt{x+1}$.

Determine and simplify the following.

(a) $(f \circ g)(x)$ (2)

(b) $(g \circ f)(x)$ (2)

6.2 Given $F(x) = \sqrt[3]{1-x^2}$

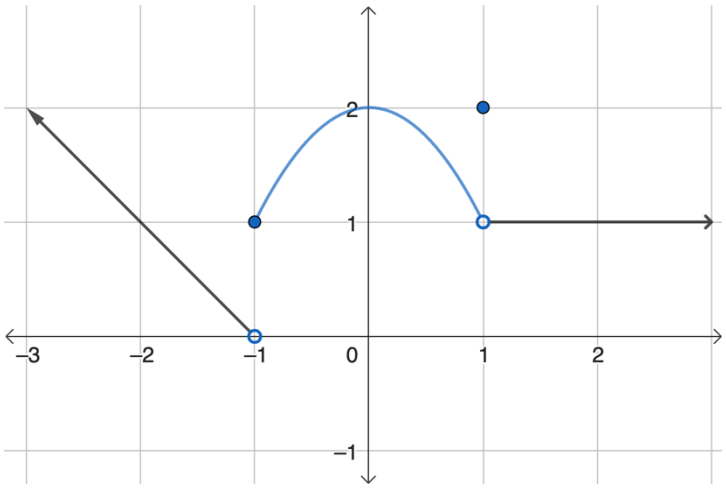
If F is defined as $F(x) = (f \circ g)(x)$, determine $f(x)$ and $g(x)$. (2)

6.3 Sketch the following piecewise functions where $x \in \mathbb{R}$. Use **DIAGRAM SHEET 2** for the sketch. (5)

$$f(x) = \begin{cases} x^2 - 2 & if \quad x < 1 \\ 2x & if \quad x = 1 \\ \dfrac{1}{x} & if \quad x > 1 \end{cases}$$

6.4. The graph shows the function:

$$f(x) = \begin{cases} a - x & \text{if} \quad x < -1 \\ 2 - x^2 & \text{if} \quad -1 \leq x < 1 \\ b & \text{if} \quad x = 1 \\ c & \text{if} \quad x > 1 \end{cases}$$



Use the graph to determine the values of a, b and c. (3)

Question 7 [20 marks]

7.1 Given the vectors $\boldsymbol{u} = (-1; 3)$ and $\boldsymbol{v} = (2; 4)$.

(a) Determine $2\boldsymbol{u} - \boldsymbol{v}$. (2)

(b) Calculate the magnitude of $\boldsymbol{u}$. (2)

(c) Calculate the direction of vector $\boldsymbol{v}$ with regards to the x-axis. (2)

7.2 Given vectors $\boldsymbol{a} = (-1; 2)$, $\boldsymbol{b} = (3; -2)$ and $\boldsymbol{c} = (3; y)$. The magnitudes of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by $|a| = \sqrt{5}$ and $|b| = \sqrt{13}$.

(a) Determine the unit vector of vector $\boldsymbol{a}$ and leave your answer in surd form. (2)

(b) Determine $a \cdot b$, the dot product of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (3)

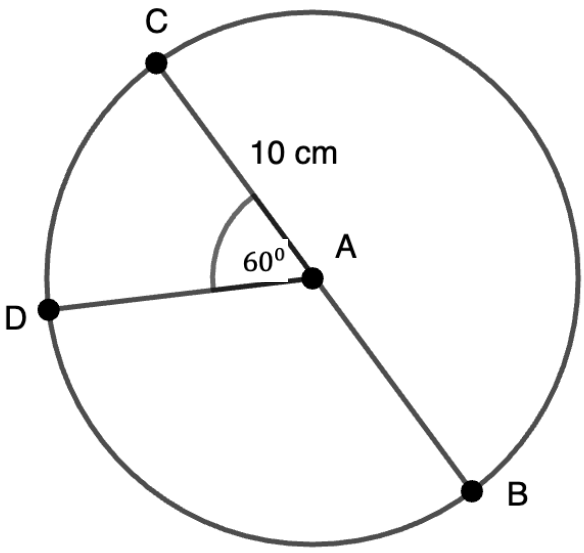
(c) Calculate the size of the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (2)

(d) Determine the value of y if the vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are at right angles with each other. (3)

7.3 In the sketch, there is a circle with diameter BC, with a radiusof $10cm$. Circle segment ACD has an angle of 60^0.

(a) Convert angle $D\hat{A}C$ to radians. (1)

(b) Determine the length of arc length BD. (3)



- END OF PAPER-

ALPHA MATHEMATICS FORMULA SHEET

MATRIXES and VECTORS:

Cramer's rule: $x_i = \frac{|A_i|}{|A|}$

$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$

$\boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y$

CALCULUS:

$V = \pi \int_a^b [f(x)]^2 \, dx$

$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$

TRIGONOMETRy:

In a sector: $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$

TABLE OF DERIVATIVES:

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)].g'(x)$

Alpha Mathematics Grade 10 – Half year exam 2024

ANSWER SHEET

Name and Surname:

_____

Question Total	1 [20]	2 [25]	3 [18]	4 [17]	5 [16]	6 [14]	7 [20]	TOTAL 130
Learner mark								

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 Question 2.4

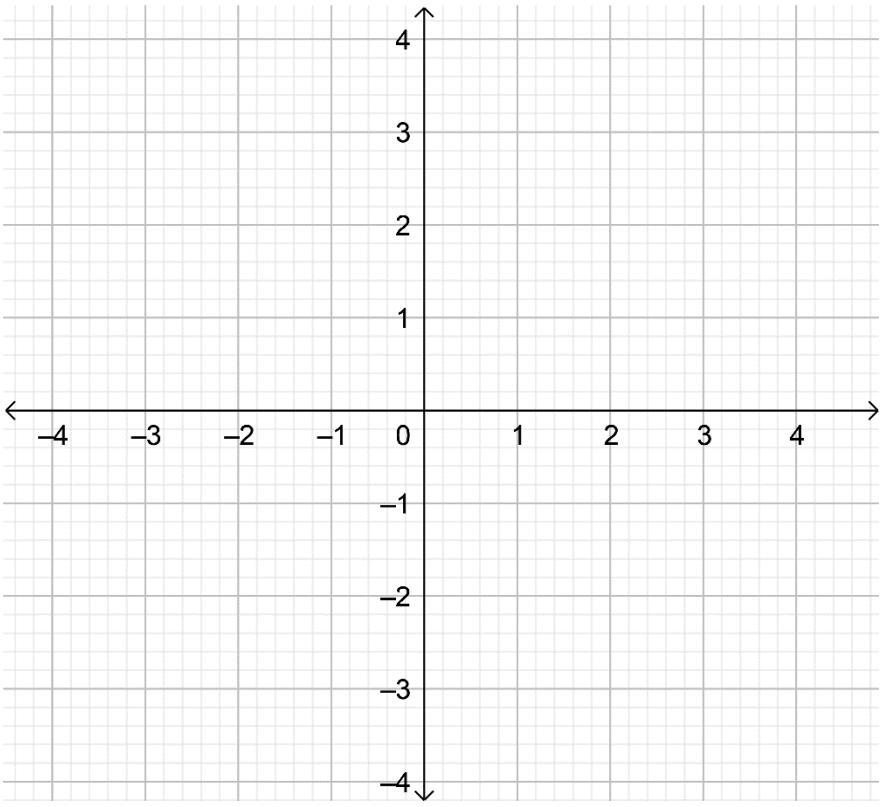


DIAGRAM SHEET 2 Question 6.3