

α -WISKUNDE/ MATHEMATICS

Rekordeksamen/ Preliminary Exam 2022
Graad/ Grade 12

Tyd/ Time: 3 uur/ hours
Totaal/ Total: 200 PUNTE/ MARKS

VRAAG/ QUESTION 1 [20 PUNTE/ MARKS]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

VRAAG/ QUESTION 2 [24 PUNTE/ MARKS]

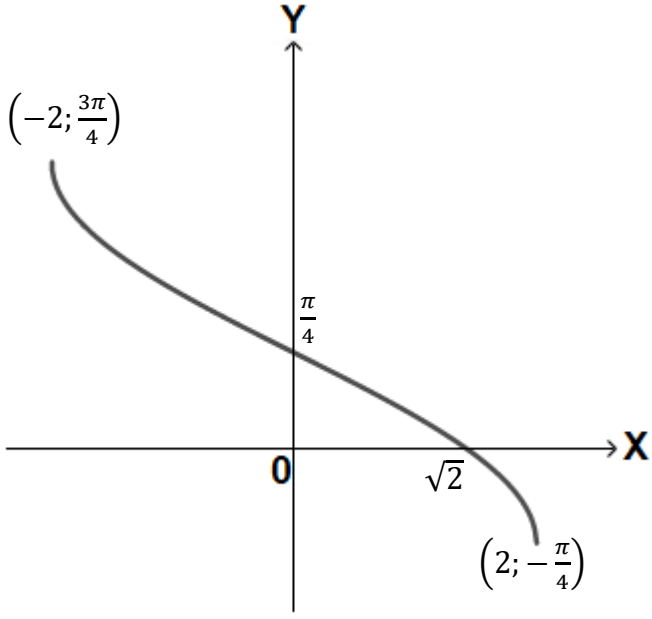
2.1a	$f(0) = 5 + 75e^{-0,2(0)} \checkmark$ $= 80^\circ\text{C} \checkmark$	1: Vervang/ <i>Substitute</i> 1: Antwoord/ <i>Answer</i> [2]
2.1b	\checkmark $15 = 5 + 75e^{-0,2t}$ $\therefore -0,2t = \ln \frac{10}{75} \checkmark$ $\therefore t = 10,07 \text{ min} \checkmark$	1: Vervang/ <i>Substitute</i> 1: Verander na/ <i>Change to ln</i> 1: Antwoord/ <i>Answer</i> [3]
2.1c	$f'(t) = (-0,2)75e^{-0,2t} \checkmark$ $= -15e^{-0,2t} \checkmark$	2: Differensieer/ <i>Differentiate</i> [2]
2.2a	$\begin{pmatrix} 2 & 4 \\ -1 & p \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \checkmark \checkmark$	2: Matriksvorm/ <i>Matrix form</i> [2]
2.2b	$\begin{vmatrix} 2 & 4 \\ -1 & p \end{vmatrix} \checkmark$ $= 2p + 4 \checkmark$ $= 0 \checkmark$ $\therefore p = -2 \checkmark$	1: Determinant van/ <i>Determinant of A</i> 1: Vereenvoudig/ <i>Simplify</i> 1: = 0 1: Antwoord/ <i>Answer</i> [4]
2.3a	\checkmark $2 \left(\cos \left(\frac{3\pi}{4} \right) + i \sin \left(\frac{3\pi}{4} \right) \right) \checkmark$	1: 2 1: $\frac{3\pi}{4}$ [2]
2.3b	\checkmark $3 \left(\cos \left(\frac{4\pi}{3} \right) + i \sin \left(\frac{4\pi}{3} \right) \right) \text{ OR/ OF } 3 \left(\cos \left(\frac{-2\pi}{3} \right) + i \sin \left(\frac{-2\pi}{3} \right) \right) \checkmark$	1: 3 1: $\frac{4\pi}{3}$ OR/ OF $\frac{-2\pi}{3}$ [2]
2.3c	\checkmark $\left(6 \text{ cis } \left(\frac{25\pi}{12} \right) \right)^4 \checkmark$ $= 1296 \text{ cis } \left(\frac{25\pi}{3} \right) \checkmark$ $= 1296 e^{\frac{25\pi}{3}i} \checkmark$ $= 648 \checkmark + 648\sqrt{3}i \checkmark \text{ OR/ OF } = 648 + 1122,37i$ OR/ OF $\left(6 \text{ cis } \left(\frac{\pi}{12} \right) \right)^4$ $= 1296 \text{ cis } \left(\frac{\pi}{3} \right)$ $= 1296 e^{\frac{\pi}{3}i}$ $= 648 + 648\sqrt{3}i$	1: 6 1: hoek/ <i>angle</i> $\frac{25\pi}{12} / \frac{\pi}{12}$ 1: 1296 1: hoek/ <i>angle</i> $\frac{25\pi}{3} / \frac{\pi}{3}$ 1: $1296 e^{\frac{25\pi}{3}i} / 1296 e^{\frac{\pi}{3}i}$ 2: Antwoord/ <i>Answer</i> [7]

VRAAG/ QUESTION 3 [26 PUNTE/ MARKS]

3.1a	$\checkmark \checkmark$ $A(-1; 4)$	2: $A(-1; 4)$	[2]
3.1b	$B(0; 1) \checkmark$	1: $B(0; 1)$	[1]
3.1c	$x^2 - 1 = -(-x - 1) + 4 \checkmark$ $\therefore x^2 - x - 6 = 0 \checkmark$ $\therefore x = 3 \checkmark$ of $x = -2 \checkmark$ n.v.t./ n/a $f(-2) = - -2 + 1 + 4 = 3$ $\therefore D(-2; 3) \checkmark$	1: Vgl/ Eq 1: Vereenvoudig/ Simplify 1: $x = 3$ 1: $x = -2$ 1: $D(-2; 3)$	[5]
3.1d	$\therefore x + 1 > 3 \checkmark$ $\therefore x + 1 < -3 \checkmark$ of/ or $x + 1 > 3 \checkmark$ $\therefore x > 2 \checkmark$ of/ or $x < -4 \checkmark$	1: Vereenvoudig/ Simplify 2: Vereenvoudig/ Simplify 2: Antwoord/ Answer	[5]
3.2	$x = 1 - i$ ook nulpunt \checkmark $\therefore (x - 1 + i)(x - 1 - i) \checkmark$ is 'n faktor/ is a factor $= x^2 - 2x + 2 \checkmark$ $\therefore g(-1) = 0 \therefore (x + 1)(x^2 - 2x + 2) \checkmark$ $g(x) = x^3 - x^2 + 2 \checkmark$	1: Ander nulpunt/ Other zero 1: faktor/ factor 1: Vereenvoudig/ Simplify 1: Vermenigvuldig met/ Multiply with $(x + 1)$ 1: Antwoord/ Answer	[5]
3.3	$n = 8 \checkmark$ $\binom{8}{r} (x)^{8-r} \left(\frac{a}{x^3}\right)^r \checkmark$ $\therefore x^{8-r} \cdot x^{-3r} = x^0$ Dus/ Thus $8 - r - 3r = 0 \checkmark$ $\therefore r = 2 \checkmark$ $\therefore \binom{8}{2} (x)^6 \left(\frac{a}{x^3}\right)^2 = 7 \checkmark$ $\therefore 28x^6 \cdot a^2 x^{-6} = 7 \checkmark$ $\therefore a^2 = \frac{1}{4}$ $\therefore a = \pm \frac{1}{2} \checkmark \checkmark$	1: n 1: Formule/ Formula 1: Stel gelyk aan 0/ Set equal to 0 1: r 1: Stel gelyk aan 7/ Equals 7 1: Vereenvoudig/ Simplify 2: Antwoord/ Answer	[8]

VRAAG/ QUESTION 4 [24 PUNTE/ MARKS]

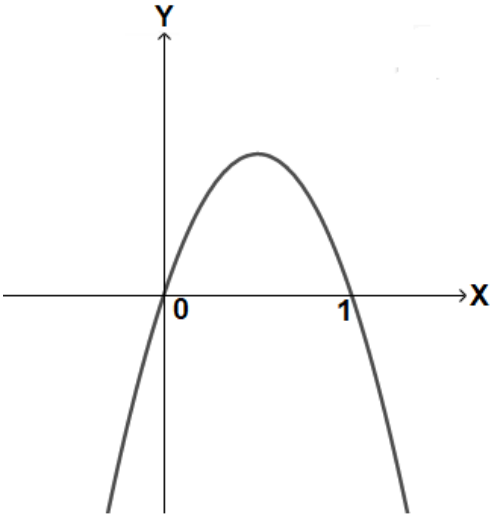
4.1	<p>Stel/ Set $n = 1$: LK/ LHS = 2 RK/ RHS = 2 \therefore Die bewering is waar as/ The statement is true for $n = 1$ ✓</p> <p>Aanvaar die bewering is waar vir/ Accept the statement is true for $n = k$: $1(2) + 2(5) + \dots + k(3k - 1) = k^2(k + 1)$ ✓</p> <p>Beskou nou/ Consider $n = k + 1$: LK/ LHS = $k^2(k + 1) + (k + 1)(3(k + 1) - 1)$ ✓ $= (k + 1)(k^2 + 3k + 2)$ ✓ $= (k + 1)(k + 1)(k + 2)$ ✓</p> <p>RK/ RHS = $(k + 1)^2(k + 1 + 1)$ ✓ $= (k + 1)^2(k + 2)$ ✓</p> <p>OF $k^3 + 4k^2 + 5k + 2$ LK = RK/ LHS = RHS en die bewering is dus waar vir $n = k + 1$/ and the statement is therefore true for $n = k + 1$. Volgens die beginsel van wiskundige induksie is die bewering dus waar vir alle $n \in \mathbb{N}$ /By the principle of mathematical induction the statement is true for all $n \in \mathbb{N}$ ✓✓✓</p>	<p>1: Bewys waar vir $n = 1$/ Prove true for $n = 1$</p> <p>1: Aanvaar waar vir/ Accept true for $n = k$</p> <p>1: Vervang/ Substitute 1: $(k + 1)$de term/ $(k + 1)$th term 2: Vereenvoudig LK Simplify LHS 1: Rk Vervang/ Substitute 1: Vereenvoudig RK/ Simplify RHS</p> <p>2: Afleiding/ Deduction</p> <p style="text-align: right;">[10]</p>
4.2a	<p>$a(2) + (-1)(-a) + (-2)(3)$ ✓ $= 3a - 6 = 0$ ✓</p> <p>$\therefore a = 2$ ✓</p>	<p>1: Formule reg/ Formula correct 1: Vereenvoudig/ Simplify $1: = 0$</p> <p>1: Antwoord/ Answer</p> <p style="text-align: right;">[4]</p>
4.2b	<p>$\begin{vmatrix} i & j & k \\ 2 & -1 & -2 \\ 2 & -2 & 3 \end{vmatrix}$ ✓ $= i(-3 - 4) - j(6 + 4) + k(-4 + 2)$ ✓ $= -7i - 10j - 2k$ ✓ en/ and $7i + 10j + 2k$ ✓</p>	<p>1: Determinant</p> <p>1: Brei uit/ Expand</p> <p>1: Antwoord/ Answer 1: Antwoord/ Answer</p> <p style="text-align: right;">[4]</p>
4.2c	<p>Oppervl/ Area = $\sqrt{7^2 + 10^2 + 2^2}$ ✓ $= 3\sqrt{17}$ of/ or $12,37$ ✓</p>	<p>1: Formule/ Formula 1: Antwoord/ Answer</p> <p style="text-align: right;">[2]</p>
4.2d	<p>$p = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{17}$ ✓</p> <p>$\theta = \text{bgcos} \frac{3}{\sqrt{17}} / \arccos \frac{3}{\sqrt{17}}$ ✓ $= 0,76$ ✓</p>	<p>1: p</p> <p>2: Formule/ Formula</p> <p>1: Antwoord/ Answer</p> <p style="text-align: right;">[4]</p>

5.1		<p>2: $(-2; \frac{3\pi}{4})$ 1: $y = \frac{\pi}{4}$ 1: $x = \sqrt{2}$ 2: $(2; -\frac{\pi}{4})$ 1: Vorm/ Form</p>
5.2a	<p>Opp/ Area = $\frac{1}{2} \cdot 2^2 \theta \checkmark$ $= 2\theta \checkmark$</p>	<p>1: Formule/ Formula 1: Antwoord/ Answer</p>
5.2b	<p>Opp/ Area = $\frac{1}{2} \cdot 2^2 (2\pi - \theta) \checkmark$ $= 4\pi - 2\theta \checkmark$</p> <p>OF/ OR Opp/ Area = $\pi \cdot 2^2 - 2\theta = 4\pi - 2\theta$</p>	<p>1: Formule/ Formula 1: Antwoord/ Answer</p>
5.2c	<p>$4\pi - 2\theta = 5(2\theta) = 10\theta \checkmark$ $\therefore \theta = \frac{\pi}{3} \checkmark$</p>	<p>1: $4\pi - 2\theta = 5(2\theta)$ 1: Antwoord/ Answer</p>
5.2d	<p>$AB = 2$ (gelyksydige/ equilateral Δ) \checkmark Boog/ Arc $AB = \frac{\pi}{3} (2) = \frac{2\pi}{3} \checkmark$ \therefore Omtrek/ Perimeter = $2 + \frac{2\pi}{3}$ of/ or $4,09 \checkmark$</p>	<p>1: AB 1: Boog/ Arc AB 1: Omtrek/ Perimeter</p>
5.3	<p>Stel/ Let $h(x) = \tan x - \frac{b}{\sqrt{1-x^2}} \checkmark$</p> <p>$\therefore h'(x) = \sec^2 x \checkmark - \frac{-1}{\sqrt{1-x^2}} \checkmark$</p> <p>$\therefore x_{n+1} = x_n - \frac{\tan x_n - \frac{b}{\sqrt{1-x_n^2}}}{\sec^2 x_n + \frac{1}{\sqrt{1-x_n^2}}} \checkmark$</p> <p>$\therefore x \approx 0.6847 \checkmark$</p>	<p>1: $h(x)$ 2: Differensieer/ Differentiate 1: Formule/ Formula 1: Antwoord/ Answer</p>

VRAAG/ QUESTION 6 [19 PUNTE/ MARKS]

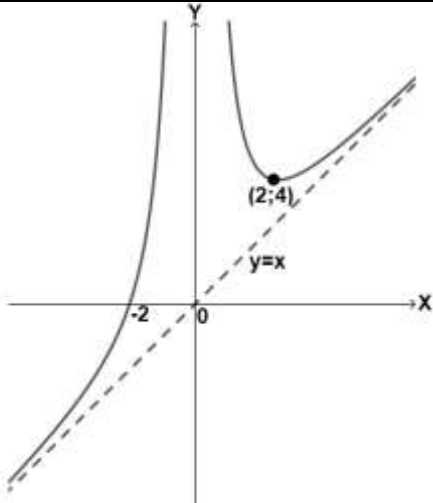
6.1a	$\lim_{x \rightarrow a^-} g(x) = 2 - a^2 \checkmark$ $\lim_{x \rightarrow a^+} g(x) = a - 4 \checkmark$ $\therefore 2 - a^2 = a - 4 \checkmark$ $\therefore a^2 + a - 6 = 0$ $\therefore (a + 3)(a - 2) = 0$ $\therefore a = -3 \text{ of/or } a = 2 \checkmark$	1: $\lim_{x \rightarrow a^-} g(x)$ 1: $\lim_{x \rightarrow a^+} g(x)$ 1: $2 - a^2 = a - 4$ 1: Antwoord/ Answer [4]
6.1b	$g'(x) = \begin{cases} -2x \frac{as}{if} x \leq 2 \\ 1 \frac{as}{if} x > 2 \end{cases}$ $\lim_{x \rightarrow 2^-} g'(x) = -2(2) = -4 \checkmark$ $\lim_{x \rightarrow 2^+} g'(x) = 1 \checkmark$ $\lim_{x \rightarrow 2^-} g'(x) \neq \lim_{x \rightarrow 2^+} g'(x) \checkmark$ $\therefore g$ is nie differensieerbaar by $x = 2$ nie / is not differentiable at $x = 2$. \checkmark	1: $\lim_{x \rightarrow 2^-} g'(x) = -4$ 1: $\lim_{x \rightarrow 2^+} g'(x) = 1$ 1: $\lim_{x \rightarrow 2^-} g'(x) \neq \lim_{x \rightarrow 2^+} g'(x)$ 1: Afleiding/ Deduction [4]
6.2a	$p'(x) = 4^{3x} \cdot \ln 4 \cdot 3 + 12x^2 + \frac{4}{4x \ln 3} \checkmark \checkmark \checkmark \checkmark \checkmark$ <p style="text-align: center;">OF/ OR</p> $p'(x) = 4^{3x} \cdot \ln 4 \cdot 3 + 12x^2 + \frac{1}{x \ln 3} \checkmark \checkmark \checkmark$	1: 4^{3x} 1: $\ln 4$ 1: 3 1: $12x^2$ 1: 4 1: $4x$ 1: $\ln 3$ [7]
6.2b	$t'(x) = 5(\text{bgtan}(2x))^4 \cdot \frac{2}{1 + (2x)^2} \checkmark \checkmark \checkmark \checkmark$	1: 5 1: $(\text{bgtan}(2x))^4$ 1: 2 1: $1 + (2x)^2$ [4]

VRAAG/ QUESTION 7 [21 PUNTE/ MARKS]

7.1a	$x < -\frac{1}{2}$ ✓ $f'(x) > 0$ as/if f styg/ increases ✓	1: $x < -\frac{1}{2}$ 1: Motivering/ Motivation [2]
7.1b	$x = -\frac{1}{2}$ ✓ en/and $x = 1$ ✓ $f'(x) = 0$ by/at f se stasionêre punte / stationary points. ✓	1: $x = -\frac{1}{2}$ 1: $x = 1$ 1: Motivering/ Motivation [3]
7.1c	 <p>Motivering: (Slegs ter verduideliking, nie nodig vir punte nie./ Motivation, not necessary for marks)</p> <p>f'' het/has x – afsnitte/intercepts by/at $x = 0$ en/and $x = 1$ (dit is f se buigpunte / that is f's inflection points) $f''(x) < 0$ as/if $x < 0$ en/and $x > 1$, dit is waar f konkaaf af buig / that is where f bends concave down. $f''(x) > 0$ as/if $0 < x < 1$, dit is waar f konkaaf op buig/ that is where f bends concave up.</p>	1: $x = 0$ 1: $x = 1$ 1: $f''(x) < 0$ as/ if $x < 0$ 1: $f''(x) > 0$ as/ if $0 < x < 1$ 1: $f''(x) < 0$ as/ if $x > 1$ [5]
7.2a	✓ ✓ ✓ ✓ ✓ $e^{xy} \left(y + x \frac{dy}{dx} \right) = \frac{1}{y} \cdot \frac{dy}{dx}$ $\therefore e^{xy} \cdot y + x e^{xy} \cdot \frac{dy}{dx} = \frac{1}{y} \cdot \frac{dy}{dx}$ $\therefore \frac{dy}{dx} = \frac{-e^{xy} \cdot y}{x e^{xy} - \frac{1}{y}}$ ✓	5: Differensieer/ Differentiate 2: Antwoord/ Answer [7]

7.2b	<p>(1; 1):</p> $\frac{dy}{dx} = \frac{-e^{1(1)} \cdot 1}{1 \cdot e^{1(1)} - 1}$ $= \frac{-e}{e - 1}$ <p>\therefore Gradiënt van normaal/ Slope of normal = $\frac{e-1}{e}$</p>	<p>2: Vervang/ Substitute</p> <p>1: Antwoord van/ Answer of $\frac{dy}{dx}$</p> <p>1: Antwoord van normaal/ Answer of normal</p> <p style="text-align: right;">[4]</p>
------	--	--

VRAAG/ QUESTION 8 [19 PUNTE/ MARKS]

8.1		<p>1: $x = -2$</p> <p>1: $y = x$</p> <p>2: (2; 4)</p> <p>1: minimum</p> <p>1: $\lim_{x \rightarrow 0^-} f(x) = \infty$</p> <p>1: $\lim_{x \rightarrow 0^+} f(x) = \infty$</p> <p>1: Vorm/ Form</p> <p style="text-align: right;">[8]</p>
-----	--	---

<p>8.2 $\Delta x_i = \frac{2-1}{n} = \frac{1}{n} \checkmark$ $x_i = 1 + \frac{i}{n} \checkmark$</p> <p>$f(x_i) = -\left(1 + \frac{i}{n}\right)^2 \checkmark = -1 - \frac{2i}{n} - \frac{i^2}{n^2} \checkmark$</p> <p>$f(x_i) \cdot \Delta x_i = -\frac{1}{n} - \frac{2i}{n^2} - \frac{i^2}{n^3} \checkmark$</p> <p>$\therefore \sum_{i=1}^n f(x_i) \cdot \Delta x_i = -\frac{1}{n} \sum_{i=1}^n 1 - \frac{2}{n^2} \sum_{i=1}^n i - \frac{1}{n^3} \sum_{i=1}^n i^2 \checkmark$</p> <p>$= -\frac{1}{n}(n) \checkmark - \frac{2}{n^2} \left(\frac{n^2}{2} + \frac{n}{2}\right) \checkmark - \frac{1}{n^3} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \checkmark$</p> <p>$= -1 - 1 - \frac{1}{n} - \frac{1}{3} - \frac{1}{2n} - \frac{1}{6n^2} \checkmark$</p> <p>$\therefore \lim_{n \rightarrow \infty} (f(x_i) \cdot \Delta x_i) = -1 - 1 - \frac{1}{3} = -\frac{7}{3} \checkmark$</p> <p>$\int_1^2 (-x^2) dx = \frac{-7}{3}$</p>	<p>1: Δx_i</p> <p>1: x_i</p> <p>2: $f(x_i)$</p> <p>1: $f(x_i) \cdot \Delta x_i$</p> <p>1: Sigma</p> <p>1: Vervang/ Substitute i^2</p> <p>1: Vervang/ Substitute i</p> <p>1: Vervang/ Substitute 1</p> <p>1: Vereenvoudig/ Simplify</p> <p>1: Antwoord/ Answer</p> <p style="text-align: right;">[11]</p>
<p>OF/ OR</p> <p>$\Delta x_i = \frac{1}{n} \checkmark$; $x_i = 1 + \frac{i}{n} \checkmark$</p> <p>$f(x_i) = -\left(1 + \frac{i}{n}\right)^2 \checkmark = -1 - \frac{2i}{n} - \frac{i^2}{n^2} \checkmark$</p> <p>$\sum_{i=1}^n \left(-1 - \frac{2i}{n} - \frac{i^2}{n^2}\right) = -\sum_{i=1}^n 1 \checkmark - \frac{2}{n} \sum_{i=1}^n i - \frac{1}{n^2} \sum_{i=1}^n i^2$</p> <p>$= -n \checkmark - \frac{2}{n} \left(\frac{n^2}{2} + \frac{n}{2}\right) \checkmark - \frac{1}{n^2} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \checkmark$</p> <p>$= -n - n - 1 - \frac{n}{3} - \frac{1}{2} - \frac{1}{6n}$</p> <p>$= -2n - \frac{3}{2} - \frac{n}{3} - \frac{1}{6n} \checkmark$</p> <p>$\int_1^2 -x^2 dx = \text{basis x hoogte}$</p> <p>$= \frac{1}{n} \times \left(-2n - \frac{3}{2} - \frac{n}{3} - \frac{1}{6n}\right) = -2 - \frac{3}{2n} - \frac{1}{3} - \frac{1}{6n^2} \checkmark$</p> <p>$\int_1^2 -x^2 dx = \lim_{n \rightarrow \infty} \left(-2 - \frac{3}{2n} - \frac{1}{3} - \frac{1}{6n^2}\right) = \frac{-7}{3} \checkmark$</p> <p>Dus $\int_1^2 (-x^2) dx = \frac{-7}{3}$</p>	

9.2	$\text{Stel/Let } f(x) = 3\ln x \text{ en/and } g'(x) = -2x + 4$ $\therefore f'(x) = \frac{3}{x} \text{ en/and } g(x) = -x^2 + 4x$ $\therefore \int (3\ln x)(-2x + 4)dx$ $= 3\ln x(-x^2 + 4x) - \int \frac{3}{x}(-x^2 + 4x)dx$ $= 3\ln x(-x^2 + 4x) + 3 \int x dx - \int 12 dx$ $= 3\ln x(-x^2 + 4x) + \frac{3}{2}x^2 - 12x$ $\therefore \int_1^2 (3\ln x)(-2x + 4)dx = 3\ln x(-x^2 + 4x) + \frac{3}{2}x^2 - 12x \Big _1^2$ $= \left[3\ln(2)(-2^2 + 4(2)) + \frac{3}{2} \cdot 2^2 - 12(2) \right] - \left[3\ln(1)(-1^2 + 4(1)) + \frac{3}{2} \cdot 1^2 - 12(1) \right]$ $= 0,82$	<p>2: f, g'</p> <p>2: f, g</p> <p>2: 1e stap/ 1st step</p> <p>2: Integreer/ Integrate</p> <p>2: Korrek invervang/ Correct substitution</p> <p>1: Antwoord/ Answer</p> <p style="text-align: right;">[11]</p>
-----	--	---