

# **$\alpha$ -MATHEMATICS**

**Alpha Wiskunde Graad 11 / *Alpha Mathematics Grade 11***

**Finale Eksamen Toets 2021 / *Final Examination 2021***

## **MEMORANDUM**

**Totaal / *Total*: 150 punte / *marks***

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**Moderator: Anna Muller**

**Hierdie memorandum bestaan uit 11 bladsye. /**

***This memorandum consists of 11 pages.***

Vraag / Question 1

[20 punte / marks]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

NR. NO	ANTWOORD ANSWER	BEREKENINGE (nie vir nasien doeleindes nie) CALCULATIONS (not for marking purpose)	PUNTE MARKS
1.1	C	$x = -2 - \sqrt{2}$	(2)
1.2	C	$9 \sin^2 3x \cdot \cos 3x$	(2)
1.3	A	$\frac{A}{(x-2)} + \frac{B}{(x-2)^2}$	(2)
1.4	C	$ba = (1 - 4; 2 - 5; 3 - 6)$ $ba = (-3; -3; -3)$	(2)
1.5	B	Perpendicular	(2)
1.6	A	$s = r\theta$ $9,4247 = x \left(\frac{\pi}{3}\right)$ $x = 8,999$ $x \approx 9$	(2)
1.7	C	$x \in [-1; 1] \ \& \ y \in [0; \pi]$	(2)

1.8	C	gradient at $x = 6$ $m = \frac{6 - 0}{4 - 10}$ $m = -1$	(2)
1.9	B	Undefined	(2)
1.10	B	The limit does not exist at $x = 4$ .	(2)

**Vraag / Question 2**

**[12 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
2.1	$\sqrt[3]{27 - x}$ $= (27 - x)^{\frac{1}{3}} \checkmark$ $= \left[ 27 \sqrt{\left( 1 - \frac{x}{27} \right)} \right]^{\frac{1}{3}}$ $= 3 \sqrt{\left( 1 - \frac{x}{27} \right)^{\frac{1}{3}}}$ $= 3 \left[ 1 + \frac{1}{3} \left( -\frac{x}{27} \right) \checkmark + \frac{\frac{1}{3} \left( \frac{1}{3} - 1 \right) \checkmark}{2!} \left( -\frac{x}{27} \right)^2 \right]$ $= 3 \checkmark - \frac{x}{27} \checkmark - \frac{x^2}{2187} \checkmark$	(8)
2.2	$\left( x + \frac{1}{x} \right)^6$ $\binom{n}{r} a^{n-r} b^r$ $= \binom{6}{r} (x)^{6-r} \left( \frac{1}{x} \right)^r \checkmark \checkmark$ $= \binom{6}{r} x^{6-r} \cdot x^{-r}$ $6 - r - r = 0 \quad \text{constant term}$ $r = 3 \checkmark$ $\binom{6}{3} x^{6-3} \cdot x^{-3}$ $= 20 \checkmark$	(4)



## Vraag / Question 4

[28 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
4.1	$\frac{7x^2 + 6x + 5}{(x + 1)(x^2 + x + 1)} \equiv \frac{A}{x + 1} \checkmark + \frac{Bx + C}{x^2 + x + 1} \checkmark$ $7x^2 + 6x + 5 = A(x^2 + x + 1) + (Bx + C)(x + 1) \checkmark$ $= Ax^2 + Ax + A + Bx^2 + Cx + Bx + C \checkmark$ $= x^2(A + B) + x(A + B + C) + A + C \checkmark$ <p>Let <math>x = -1</math>  <math>6 = A \checkmark</math></p> <p><math>A + B = 7</math>  <math>B = 1 \checkmark</math></p> <p><math>A + C = 5</math>  <math>C = -1 \checkmark</math></p> $\frac{7x^2 + 6x + 5}{(x + 1)(x^2 + x + 1)} \equiv \frac{6}{x + 1} + \frac{x - 1}{x^2 + x + 1} \checkmark$	(9)

4.2

 $x = 2 - 5i$  is also a root of  $P(x)$  ✓

$$(x - 2 - 5i)(x - 2 + 5i) ✓$$

$$= (x^2 - 4x + 29)$$

 $(x^2 - 4x + 29)$  is a factor of  $P(x)$  ✓✓

$$\begin{array}{r}
 x^2 \quad -4x \quad +29 \quad \left| \begin{array}{r}
 x^2 \quad -x \quad -2 \\
 \hline
 x^4 \quad -5x^3 \quad +31x^2 \quad -21x \quad -58 \\
 x^4 \quad -4x^3 \quad +29x^2 \\
 \hline
 \phantom{x^4} \quad -x^3 \quad +2x^2 \quad -21x \quad -58 \\
 \phantom{x^4} \quad -x^3 \quad +4x^2 \quad -29x \\
 \hline
 \phantom{x^4} \phantom{-x^3} \quad -2x^2 \quad +8x \quad -58 \\
 \phantom{x^4} \phantom{-x^3} \quad -2x^2 \quad +8x \quad -58 \\
 \hline
 \phantom{x^4} \phantom{-x^3} \phantom{-2x^2} \phantom{+8x} \quad 0
 \end{array}
 \right.
 \end{array}$$

$$x^4 - 5x^3 + 31x^2 - 21x - 58 = 0$$

$$(x^2 - 4x + 29)(x^2 - x - 2) ✓✓✓ = 0$$

$$(x^2 - 4x + 29)(x + 1) ✓(x - 2) ✓ = 0$$

$$x = -1 ✓ ; 2 ✓$$

(11)

4.3	$LHS = \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \left(1 - \frac{1}{16}\right) \dots \left(1 - \frac{1}{k^2}\right) \left(1 - \frac{1}{(k+1)^2}\right)$ $= \left(\frac{k+1}{2k}\right) \sqrt{\left(1 - \frac{1}{(k+1)^2}\right)}$ $= \left(\frac{k+1}{2k}\right) \left(\frac{k^2 + 2k + 1 - 1}{(k+1)^2}\right) \sqrt{\phantom{x}}$ $= \frac{k^2 + 2k}{2k(k+1)}$ $= \frac{k(k+2)}{2k(k+1)} \sqrt{\phantom{x}}$ $= \frac{k+2}{2(k+1)} \sqrt{\phantom{x}}$ $RHS = \frac{k+1+1}{2(k+1)} \sqrt{\phantom{x}}$ $= \frac{k+2}{2(k+1)}$ $LHS = RHS \checkmark$ <p>The statement is true for <math>n = 2</math>. If it is true for <math>n = k</math>, it is also true for <math>n = k + 1</math> and consequently, true for all <math>n \geq 2, n \in \mathbb{N}</math>. <math>\checkmark</math></p>	<b>(8)</b>
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**Vraag / Question 5**

**[15 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
5.1.1		<b>(5)</b>
5.1.2	$k(x) = -2 \sqrt{\arctan\left(x + \sqrt{\frac{\pi}{2}}\right)} - 2 \sqrt{\phantom{x}}$	<b>(4)</b>

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
5.1.3	$x = 2 \arctan y \checkmark$ $\frac{x}{2} = \arctan y \checkmark$ $y = \tan \frac{x}{2} \checkmark$	(3)
5.1.4	$h'(x) = \frac{2}{1+x^2} \checkmark$ $h'(1) = \frac{2}{1+1^2} \checkmark$ $h'(1) = 1$  The gradient of $h$ at $x = 1$ is 1. $\checkmark$	(3)

**Vraag / Question 6**

**[12 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
6.1	$u = i + 2j - k$ , and $w = 5i - j + 3k$ $u \times w = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 5 & -1 & 3 \end{vmatrix} \checkmark$ $= [2(3) - (-1)(-1)]i \checkmark - [1(3) - 5(-1)]j \checkmark + [1(-1) - 2(5)]k \checkmark$ $= 5i - 8j - 11k \checkmark$  Area of parallelogram = $ u \times w $ $= \sqrt{5^2 + 8^2 + 11^2} \checkmark$ $= 14,49 \text{ units}^2 \checkmark$	(7)

6.2	$P = 3i + 5j - 4k$ $ P  = \sqrt{9 + 25 + 16} = 5\sqrt{2} \checkmark$ $Q = 2i + 4j + 3k$ $ Q  = \sqrt{4 + 16 + 9} = \sqrt{29} \checkmark$ $P \cdot Q = 3(2) + 5(4) + (-4)(3)$ $P \cdot Q = 14 \checkmark$ $14 = 5\sqrt{2} \cdot \sqrt{29} \cdot \cos \theta \checkmark$ $\cos \theta = \frac{14}{5\sqrt{2} \cdot \sqrt{29}}$ $\theta = 1,19 \text{ rad} \checkmark$	(5)
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**Vraag / Question 7**

**[11 punte / marks]**

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
7.1	$f(x) = \int (-3x^2 + 3) dx \checkmark$ $= \frac{-3x^3}{3} \checkmark + 3x \checkmark + k \checkmark$ From the diagram $k = 4$ Hence; $f(x) = -x^3 + 3x + 4 \checkmark$	(5)

7.2	$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$ $= a_n - \frac{-a_n^3 + 3a_n + 4}{-3a_n^2 + 3}$ $n \rightarrow a_n$ $1 \rightarrow 2,195833333$ $2 \rightarrow 2,195823346$ $3 \rightarrow 2,195823345$ $4 \rightarrow 2,195823345$ $5 \rightarrow 2,195823345$ $\therefore x \approx 2,19582$	(6)
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**Vraag / Question 8**

[10 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
8.1	$D_x \left[ \frac{2x^4 - 8x^2}{x^5 - 6x} \right]$ $= \frac{(8x^3 - 16x)(x^5 - 6x) - (2x^4 - 8x^2)(5x^4 - 6)}{(x^5 - 6x)^2}$	(5)
8.2	$f(x) = \cos(x \cdot \sin x)$ $f'(x) = -\sin(x \cdot \sin x) [1 \cdot \sin x + x \cdot \cos x]$	(5)

**Vraag / Question 9**

[9 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
9.1	$\int \frac{2}{\sqrt{9-x^2}} dx$ $= 2 \int \frac{1}{\sqrt{9\left(1-\left(\frac{x}{3}\right)^2\right)}} dx$ $= \frac{2}{3} \arcsin\left(\frac{x}{3}\right) + k$	(5)
9.2	$\int (6\operatorname{cosec} 4x \cdot \cot 4x + 7 \sec^2(3x + 2)) dx$ $= \frac{6(-\operatorname{cosec}^2 4x)}{4} + \frac{7 \tan(\sqrt{3x + 2})}{3} + k$	(4)

Vraag / Question 10

[9 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
10	$V = \pi \int_0^1 \left( (x^2 + 2)^2 \checkmark - \left( \frac{1}{2}x + 1 \right)^2 \checkmark \right) dx$ $V = \pi \int_0^1 \left( x^4 + 4x^2 + 4 - \frac{1}{4}x^2 - x - 1 \right) dx$ $V = \pi \int_0^1 \left( x^4 + \frac{15}{4}x^2 - x + 3 \right) \checkmark dx$ $V = \pi \left[ \frac{x^5}{5} \checkmark + \frac{15}{4} \cdot \frac{x^3}{3} \checkmark - \frac{x^2}{2} \checkmark + 3x \checkmark \right]_0^1$ $V = \pi \left[ \frac{1}{5} + \frac{5}{12} - \frac{1}{2} + 3 \right] \checkmark$ $V = \frac{79}{20} \pi$ $V \approx 12,41 \text{ units}^2 \checkmark$	(9)

Vraag / Question 11

[8 punte / marks]

NR. NO	ANTWOORD / ANSWER	PUNTE / MARKS
11	$\lim_{x \rightarrow 1^-} g(x) = 9 - 1 = 8 \checkmark$ $\lim_{x \rightarrow 1^+} g(x) = 2k \checkmark$ $2k = 8 \checkmark$ $k = 4 \checkmark$ $\lim_{x \rightarrow -1^-} g(x) = -2 + 3c \checkmark$ $\lim_{x \rightarrow -1^+} g(x) = 9 - 1 = 8 \checkmark$ $-2 + 3c = 8 \checkmark$ $3c = 10$ $c = \frac{10}{3} \checkmark$	(8)

- EINDE VAN DIE MEMORANDUM / END OF THE MEMORANDUM -