α-MATHEMATICS

Grade 11 Alpha Mathematics

June Examination 2021

Examiner: L Liebenberg **Time: $2\frac{1}{2}$ hours**

Moderator: A Muller **Total: 150**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 7 pages and an answer sheet.
2. Answer ALL 6 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 **[20 marks]**

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer. Mark the correct answer with an **X** on the answer sheet.

1.1 The coordinates of the y-intercept of the function $y = |x - 3| + 2$ is (2)

A $(3\,;2)$

B $y = 5$

C $(0\,;-1)$

D $(0\,;5)$

1.2 $\sum_{k=1}^{n} k =$ (2)

A $k + k^2 + k^3 + \cdots + k^n$

B $1 + 2 + 3 + \cdots + n$

C $k + 2k + 3k + \cdots + nk$

D None of the above

1.3 The graph of $y = arcsinx$ is translated down by $\frac{\pi}{2}$ units and 2 units to the right. The new equation of the graph is (2)

A $y = \arcsin(x + 2) - \frac{\pi}{2}$

B $\mathrm{y} = \arcsin(x - 2) + \frac{\pi}{2}$

C $y = \arcsin(x - 2) - \frac{\pi}{2}$

D $\mathrm{y} = -\frac{\pi}{2}\arcsin(x - 2)$

1.4 The value in the 5th position in row 11 of Pascal's triangle is (2)

A 330

B 462

C 252

D 495

1.5 The inverse function of $f(x) = cos2x + 3$ is (2)

A $f^{-1}(x) = \arccos(2x + 3)$

B $f^{-1}(x) = \frac{1}{2}\arccos(x - 3)$

C $f'(x) = \frac{1}{2}\arccos(x - 3)$

D $f^{-1}(x) = \arccos(2x - 3)$

1.6 The following statement will be proven by mathematical induction in **question 4.1** (2)

$$2 + 9 + 16 + \cdots + (7n - 5) = \frac{n(7n - 3)}{2}$$

Use this statement to determine the value of:

$$2 + 9 + 16 + \cdots + 79$$

A 21725

B 161

C 335

D 486

1.7 Given that $|x| \leq 0$ choose the correct statement below (2)

A There are no solutions.

B $x \in \mathbb{R}$

C There is one solution.

D None of the above.

1.8 $(3x + 2)$ is a factor of (2)

A $x^2 + 2x - \frac{2}{3}$

B $3x^3 + 2x^2 + 1$

C $3x^3 + 5x^2 + 2x$

D $3x^3 + 2x^2 + \frac{2}{3}$

1.9 Given $h(x) = arctanx$ the domain and range will be as follows (2)

A Domain: $x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ Range: $y \in (-\infty; \infty)$

B Domain: $x \in (0; \pi)$ Range: $y \in (-1; 1)$

C Domain: $x \in (-1; 1)$ Range: $y \in (0; \pi)$

D Domain: $x \in (-\infty; \infty)$ Range: $y \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

1.10 The number of terms in the expansion of $(x - 1)^{13}$ is: (2)

A 14

B 13

C 12

D 26

Question 2 **[37 marks]**

2.1 Given the functions $f(x) = |2x - 4| - 1$ and $g(x) = x + 1$.

2.1.1 Determine $f(0)$ (2)

2.1.2 Determine the intercepts of $f(x)$ with the axes. (5)

2.1.3 Determine the coordinates of the salient point of f. (2)

2.1.4 Solve for x if $f(x) = g(x)$. (6)

2.1.5 Determine the values of x where $f(x) \leq 3$. (5)

2.1.6 Sketch the graphs of f and g on the DIAGRAM SHEET provided. (6)

2.2 Solve for x if $3|4 - x| > 15$ and $x^2 \leq 121$ (11)

Question 3 [23 marks]

3.1 Given $k(x) = x^4 - 5x^3 + 31x^2 - 21x - 58$ and has a root at $x = 2 + 5i$.

3.1.1 Determine a quadratic factor of k. (4)

3.1.2 Factorise $k(x)$ fully over $\mathbb{R}[x]$. (6)

3.1.3 Factorise $k(x)$ fully over $\mathbb{C}[x]$. (3)

3.2 Given $p(x) = 6x^3 - 7x^2 + 1$

3.2.1 Determine if $(2x - 1)$ is a factor of $p(x)$. (3)

3.2.2 Solve for x given that $p(x) = 0$. (7)

Question 4 [28 marks]

4.1 Use Mathematical Induction to prove that the following statement is true for all $n \in \mathbb{N}$: (13)

$$2 + 9 + 16 + \cdots + (7n - 5) = \frac{n(7n - 3)}{2}$$

4.2 Decompose (9)

$$\frac{4x^3 + 6x^2 + x + 2}{x^4 + x^2}$$

into partial fractions.

4.3 Match **COLUMN B** to **COLUMN A**. Write the letter in **COLUMN B** next to the matching number in **COLUMN A** on the ANSWER SHEET provided.

	COLUMN A	**COLUMN B**	
	Expression	Form of partial fraction	
4.3.1	$\frac{f(x)}{(x+1)^3}$	**A** $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$	(2)
4.3.2	$\frac{f(x)}{x^2(x^2+1)}$	**B** $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$	(2)
4.3.3	$\frac{f(x)}{x^3+2x^2+x}$	**C** $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x^2+1)}$	(2)

Question 5 [21 marks]

5.1 Given the Binomial Theorem:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r$$
$$= \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

5.1.1 Use the Binomial Theorem to determine the first three terms in the expansion of $\left(ax + \frac{1}{2y}\right)^{10}$. (6)

5.1.2 Given that term five is (7)

$$\frac{76545x^6}{8y^4}$$

determine the value of a.

5.2 Make use of the power range (8)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

to expand $\sqrt[4]{16-x}$ up to the term that contains x^3.

Question 6 [21 marks]

6.1 Given $f(x) = 3\arcsin(5x+1)$.

6.1.1 Determine the inverse, that is f^{-1}. (5)

6.1.2 Solve for x if $f(x) = \frac{\pi}{2}$. (6)

6.2 Given $g(x) = 2\arctan x$

6.2.1 Sketch the graph of g on the DIAGRAM SHEET provided. (6)

6.2.2 Describe the transformation that takes place from g to h if $h(x) = \arctan(x-1)$. (4)

- END OF QUESTION PAPER -

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Grade 11 Alpha Mathematics

June Examination 2021 Answer sheet

Name and Surname: _____

Question Total	1 [20]	2 [37]	3 [23]	4 [28]	5 [21]	6 [21]
Learner mark						

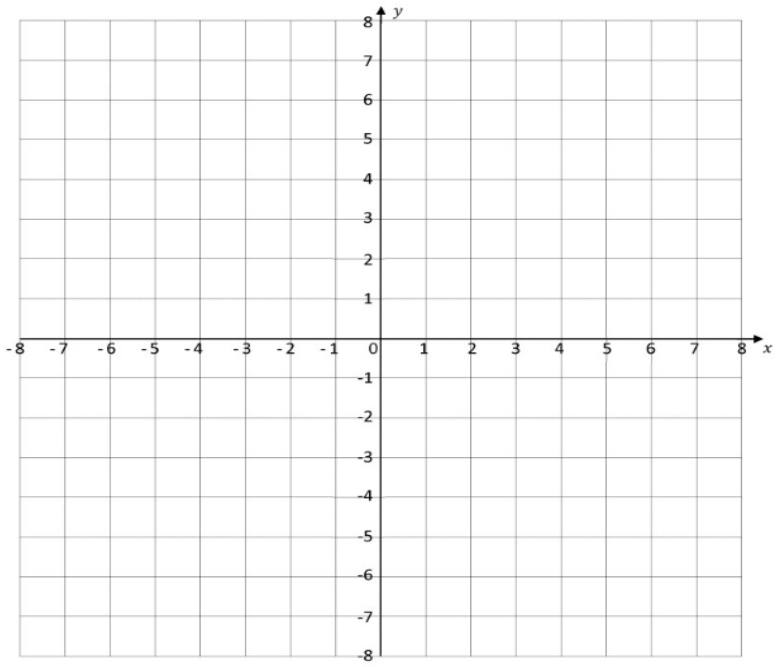
TOTAL 150

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

Question 2

2.1.6



Question 4

4.3.1	
4.3.2	
4.3.3	

Question 6

6.2.1