

α -WISKUNDE

Alpha Mathematics Half year exam 2021

Grade 12

Examiner: Pieter of Onselen

Moderator: Anna Muller

Time: 3 hours

Total: 200 points

INSTRUCTIONS EN INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 8 pages and a formula sheet of 3 pages and an answer sheet of 2 pages.
2. Answer ALL 8 questions.
3. Use the same numbering for the answers if it appears in the questions.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to **two decimal places**.
6. Except for question one, **clearly show all calculations**, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1**[20 points]**

This question must be answered **on the answer sheet**. Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 The power series $\left(1 - \frac{x}{3}\right)^{-5}$ converges when:

(A) $|x| > 3$

(B) $|x| > -3$

(C) $|x| < 3$

(D) $|x| < -3$

1.2 Determine the horizontal asymptote, if any, for $\frac{2x^2+x}{x^3-2x^2+1}$.

(A) $y = 2$

(B) $y = \frac{1}{2}$

(C) $y = 0$

(D) No horizontal asymptote.

1.3 Solve for x if $-|x - 1| > -2$

(A) $x > 3$ or $x < -1$

(B) No solution

(C) $-1 < x < 3$

(D) $x \in \mathbb{R}$

1.4 Given $f(x) = 2e^{3x}$. The inverse of f in the form $f^{-1}(x) = \dots$

(A) $3 \ln\left(\frac{x}{2}\right)$

(B) $\frac{1}{3} \ln x - \frac{1}{3} \ln 2$

(C) $\ln\left(\frac{x}{2}\right)^3$

(D) $\frac{1}{3} \ln(2x)$

1.5 Which of the following defines a jump discontinuity at $x = a$?

(A) $\lim_{x \rightarrow a} f(x)$ does not exist

(B) $f(a)$ does not exist

(C) $\lim_{x \rightarrow a} f(x) \neq f(a)$

(D) $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$

1.6 The inverse of $f(x) = \sin\left(\frac{x}{2}\right)$ will be defined if the domain of f is:

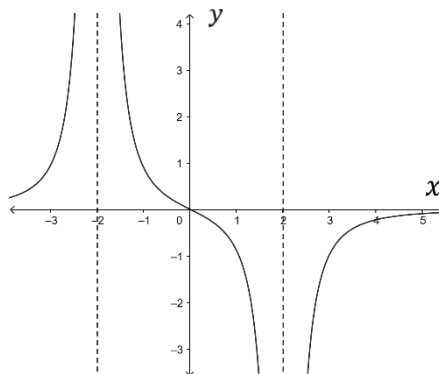
(A) $x \in \left[-\frac{\pi}{4}; \frac{\pi}{4}\right]$

(B) $x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

(C) $x \in [-\pi; \pi]$

(D) $x \in [-2\pi; 2\pi]$

- 1.7 Simplify $2(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}) \div (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$
- (A) 2 (B) 1
(C) $-2i$ (D) $2i$
- 1.8 The second term of the power series $\sqrt{4-x}$ is
- (A) $\frac{1}{4}x$ (B) $\frac{1}{8}x$
(C) $-\frac{1}{8}x$ (D) $-\frac{1}{4}x$
- 1.9 Given $f(x) = \log_3 e^x$, then $f'(x) =$
- (A) $\frac{1}{\ln 3}$ (B) $\frac{1}{e^x \ln 3}$
(C) $\frac{1}{e^x}$ (D) $\frac{e^x}{\ln 3}$
- 1.10 The sketch below shows the graph of a rational function $f'(x)$



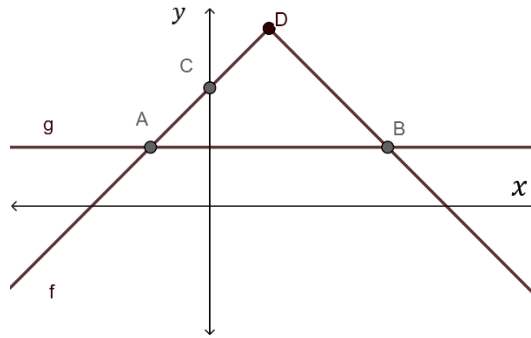
Which one of the following statements is true?

- (A) $f(x)$ is decreasing for $x > 2$
 (B) $f(x)$ has a turning point at $x = 0$
 (C) $f(x)$ has a vertical asymptote at $x = -2$ and $x = 2$
 (D) $f(x)$ is concave down between $-2 < x < 2$

Question 2**[34 points]**

2.1 The sketch below shows the graphs of $f(x) = -|x - 1| + 3$ and $g(x) = 1$.

- (a) Determine the coordinates of D , the vertex of $f(x)$. (2)
- (b) Determine the x -coordinates of points A , B and the coordinates of C . (4)
- (c) Determine subsequently graphically the values of x for which $g(x) - f(x) \geq 0$. (2)
- (d) Determine the values of x for which $f(2x) = x$. (6)



2.2 The electricity of a spaceship is generated by a type of nuclear power generator. The electricity generated can be modulated by:

$$P(t) = \frac{150}{e^{0,0078t}}$$

$P(t)$ represents the power in watts and t is the time in years that elapsed.

- (a) Determine the initial electrical power that was generated. (2)
- (b) Determine the amount of power that will be generated after 4 years. (2)
- (c) Determine how long it will take for the power generation to decrease by 20%? (5)
- (d) Determine an expression for the rate of power generated. (2)
- 2.3 Given $f(x) = x^4 - x^3 + 6x^2 + 14x - 20$ and $(x - 1 + ai)$ is a factor of $f(x)$.
- (a) If $x^2 - 2x + 10$ a factor is of $f(x)$. Determine all the values of a , $a \in \mathbb{R}$, using the **complex conjugate root theorem**. (4)
- (b) Expand $f(x)$ in factors of $\mathbb{C}[x]$. (5)

Question 3**[28 points]**

3.1 Given $\left(x^3 - \frac{3}{x}\right)^{15}$.

(a) Determine the seventh term in the expansion. (4)

(b) Determine if the expansion has a constant term (x^0).

Show all calculations and motivate your answer. (3)

3.2 Given $a = 1 - i$ and $b = \sqrt{3} + i$.

(a) Express a and b in polar form. Give the answer as $rcis\theta$ and leave r in surd form and θ in terms of π . (4)(b) Use **De Moivre's theorem** and determine the value of $(a)^3(b)^2$, give your answer in polar form. (5)

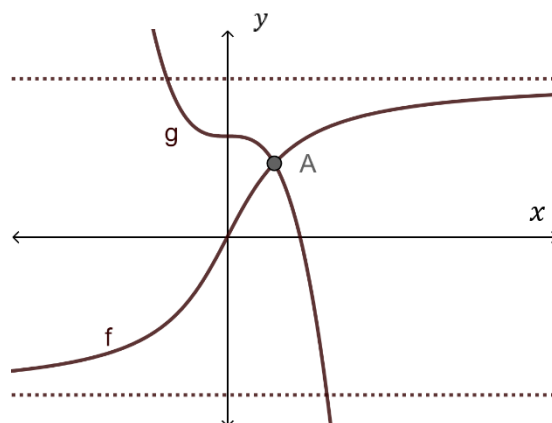
(c) Convert your answer to rectangular form. (2)

3.3 Use the Mathematical induction and prove that the following expression is true for all $n \in \mathbb{N}$.

$$1 - 3 + 9 - \dots + (-3)^{n-1} = \frac{1 - (-3)^n}{4} \quad (10)$$

Question 4**[14 points]**4.1 Use implicit differentiation and determine $\frac{dy}{dx}$ if:

$$e^{2x+3y} = x^2 - \ln(xy^3) \quad (8)$$

4.2 The sketch shows the graph $f(x) = \arctan(2x)$ and $g(x) = -3x^3 + 1$.Use the **Newton-Raphson method** and determine the x -coordinate of A the point of intersection of the functions f and g (accurate to 4 decimal places). Use $x_0 = 0,5$ as the first estimation. (6)

Question 5**[40 points]**

$$5.1 \quad f(x) = \begin{cases} x^2 + 3x + p & \text{as } x \leq 1 \\ qx + 2 & \text{as } x > 1 \end{cases}$$

Find the values of p and q if $f(x)$ is differentiable for all values of x . (7)

5.2 Determine the following derivatives. There is no need to simplify the answers.

(a) $g'(x)$ if $g(x) = 7^{\cos x}$ (3)

(b) $\frac{dy}{dx}$ if $y = \log_2(x^3) \times \ln|2x|$ (5)

(c) $Dx \left[\frac{\sin(3x)}{\arccos\left(\frac{x}{3}\right)} \right]$ (6)

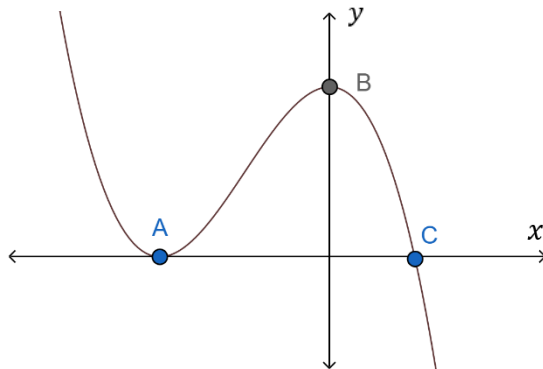
5.3 Determine the following integrals.

(a) $\int \frac{1}{\left(\frac{x}{2}\right)^{\ln 5}} dx$ (2)

(b) $\int \left(\frac{3}{4x} - \frac{4x}{3} \right) dx$ (2)

(c) $\int \left(\frac{1}{x^2 + 2x + 2} \right) dx$ (3)

5.4 Given the sketch of the function $f'(x) = -(2x - 1)(x + 1)^2$, the **derivative** of $f(x)$.



(a) Determine the coordinates of A , B and C . (5)

(b) Determine the x -values of the point(s) of inflection of $f(x)$ using the derivatives and also show that they are points of inflection. (6)

(c) For which values of x will $f(x)$ be decreasing? (2)

Question 6

[9 points]

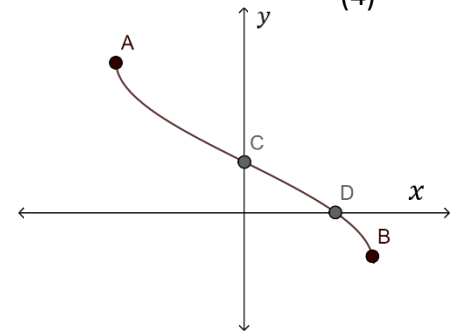
6.1 Given that $\frac{3x^2+px+2}{(x^2+1)(x-1)} \equiv \frac{x}{x^2+1} + \frac{A}{x-1}$

Determine the values of A and p by decomposing the function in partial fractions. (5)

6.2 The sketch below is the graph of $f(x) = \arccos\left(\frac{x}{2}\right) + q$. (4)

The graph intercepts the y -axis at $C\left(0; \frac{\pi}{4}\right)$.

- (a) Determine the value of q .
- (b) Determine the coordinates of A, B and D .



[21 points]

Question 7

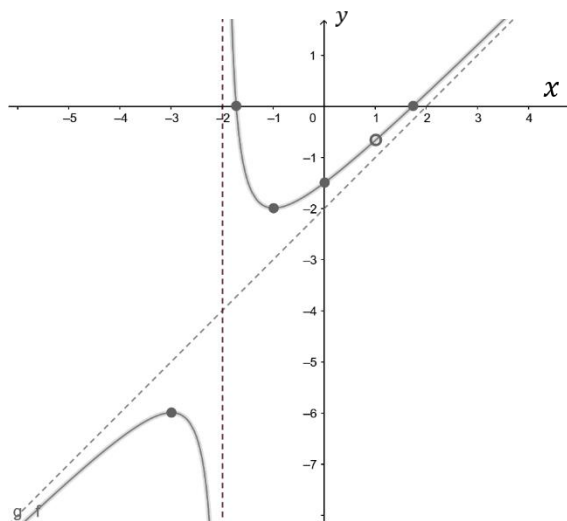
7.1 Given the function $f(x) = \frac{2x^2-4}{x^2-4x}$.

- (a) Determine the values of the x -intercept(s). (2)
- (b) Determine the equations of the asymptotes and name the asymptotes. (3)
- (c) Sketch the rational function. Use **DIAGRAM SHEET 1** for the sketch. (6)

Show all intercepts with the axes and all the equations of the asymptotes clearly on the sketch.

- (d) The graph $f(x)$ intercepts one of the asymptotes. Determine the coordinates of the intercept. (4)

7.2 The sketch shows the graph of $f(x) = x + a + \frac{x+b}{(x+c)(x+b)}$



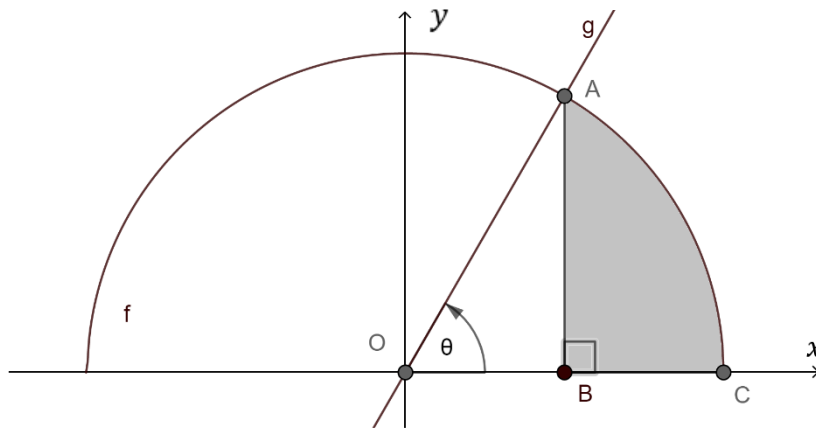
Give the values of a, b and c and motivate each answer. (6)

Question 8**[34 points]**

8.1 Given vectors $u = (a; 1; -2)$ and $v = (-2; 4; 1)$.

- (a) If $|u| = 3$ determine a . (3)
- (b) Assume that $a = 2$, determine the angle between u and v . (4)
- (c) Determine a vector that will be perpendicular to vectors u and v . (7)
- (d) Hence determine the area of a parallelogram formed by the two vectors u and v . (2)

8.2 The sketch below shows a half circle $f(x) = \sqrt{16 - x^2}$ and a straight line $g(x) = \sqrt{3}x$. The angle between the x -axis and $g(x)$ is θ .



- (a) Determine the x -coordinate of A the intercept between $f(x)$ and $g(x)$. (4)
- (b) Determine θ and OA the radius of the semi-circle.
(Hint the circle formula is $x^2 + y^2 = r^2$) (3)
- (c) Assume $\theta = \frac{\pi}{3}$ and $OA = 4$ and determine the area of the shaded area ABC . (5)
- (d) Determine the volume of revolution if the area $f(x) - g(x)$ rotates around the x -axis between $x = 0$ and $x = 2$. (6)

- END OF EXAM PAPER-

ANSWER SHEET
Alpha Mathematics Grade 12
Half year exam 2021

Name and Surname: _____

Question Total	1 [20]	2 [34]	3 [28]	4 [14]	5 [40]	6 [9]	7 [21]	8 [34]	TOTAL 200
Learner marks									

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1

