α-WISKUNDE

Alpha Wiskunde / Alpha Mathematics

Eindeksamenvraestel Memo / Exam Memo

Eksamen 2020 / Exam 2020

Graad / Grade 11

Eksaminator: AM Muller **Totaal / Total: 165 punte**

Moderator: R Grobler

Vraag 1 [20 punte]

No.	Antwoord/ Answer	Berekeninge (Nie vir nasien doeleindes nie)/ Calculations	Punte/ Marks
1.1	D ✓✓	$\frac{-2}{\|4-2x\|} \geq -4$, ½≤ $\|4-2x\|$ $4-2x \leq -½ \quad of \quad 4-2x \geq ½$ x≥ $\frac{9}{4}$ of x≤ $\frac{7}{4}$	2
1.2	C ✓✓	bgtan2x=$\frac{\pi}{4}$, 2x=tan$\frac{\pi}{4}$, 2x=1, x=½	2
1.3	C ✓✓	11 terme/11 terms	2
1.4	A ✓✓	$(2+5x)^{-3}$ =$[2(1+\frac{5x}{2})]^{-3}$ ∴ $\|\frac{5x}{2}\| < 1$, $\|x\| < \frac{2}{5}$	2
1.5	B ✓✓	x=3bgsin(y+½) – 5 $\frac{x+5}{3}$=bgsin(y+½) y+½ = sin($\frac{x+5}{3}$), y=sin($\frac{x+5}{3}$) - ½	2
1.6	D ✓✓		2
1.7	A ✓✓	boogACB =$\frac{5\pi}{3}$(5) = $\frac{25\pi}{3}$ =26,18	2
1.8	A ✓✓	15-2 +2a=15, 2a=2 : a=1	2
1.9	B ✓✓	$\lim_{x\to 3^-} f(x) = 9-a \; en \; \lim_{x\to 3^+} f(x) = 8$ 9-a=8: a=1	2
1.10	D ✓✓	$0 \leq 2x-3 \leq \pi$;3≤ $2x \leq \pi+3$ $\frac{3}{2} \leq x \leq \frac{\pi+3}{2}$: $\frac{3}{2} \leq x \leq 3{,}07$	2

Vraag 2 [11 punte]

No.	Antwoord/ Answer	Punte/ Marks
2.1	$x \geq 1$ of $x < 1$ $x-1=\frac{x-5}{3}$ ✓ $-(x-1)=\frac{x-5}{3}$ ✓ $3x-3=x-5$ $-3x+3=x-5$ X=-1✓ N.v.t. ✓ x=2✓ N.v.t✓ GEEN OPLOSSING✓	7
2.2	y=-2(x-3) +4 (a) (3;4) ✓ (b) y=-2(-3)+4 y=-2✓ (c) -2|x-3| +4=0 |x-3|=2 x-3=2 of x-3=-2 : x=5 ✓ of x=1✓	4

Vraag 3 [16 punte]

No.	Antwoord/ Answer	Punte/ Marks
3.1	(fog)(x) = f(sin($\pi - 3x$)) =1- $(sin(\pi - 3x))^2$✓ =cos^2($\pi - 3x$) ✓ =(-cos3x)^2 =cos^2(3x) ✓	3
3.2 (a)	f(x) =$\frac{3x+5}{(1-x)(1+3x)}$ =$\frac{A}{1-x}+\frac{B}{1+3x}$✓ 3x+5 = A(1+3x) + B(1-x) ✓ x=1: 8=4A A=2✓ x= -⅓ : 4 = $\frac{4B}{3}$ B = 3✓ f(x) = $\frac{2}{1-x}+\frac{3}{1+3x}$✓	5
3.2 (b)	f(x) = 2(1-x)$^{-1}$ + 3(1+3x)$^{-1}$✓ =2(1 + (-1)(-x) + $\frac{(-1)(-2)(-x)^2}{2!}$) ✓+3(1+(-1)(3x) + $\frac{(-1)(-2)(3x)^2}{2!}$) ✓ =2(1+x+x^2)✓ +3(1-3x+9x^2) ✓ =2+2x+2x^2 + 3-9x+27x^2 = 5 ✓– 7x✓ +29x^2 ✓	8

Vraag 4 [13 punte]

No.	Antwoord/ Answer	Punte/ Marks
4	x= -1 -2$\sqrt{3}$ x= -1$\pm$2$\sqrt{3}$ ✓ $(x+1)^2 = (\pm 2\sqrt{3})^2$✓ x^2 +2x + 1 = 12 x^2 +2x - 11 = 0 x^2 +2x – 11 is 'n faktor/ is a factor✓ Doen Langdeling / Do long division f(x) =(x^2 +2x – 11)(x^3✓ + 3x^2✓ +3x✓ +2✓) =(x^2 +2x – 11)(x+2) ✓ (x^2 +x + 1) ✓ =(x+1-2$\sqrt{3}$) ✓ (x+1+2$\sqrt{3}$) ✓ (x+2)(x+½ - $\frac{\sqrt{3}}{2}$ i) ✓ (x + ½ + $\frac{\sqrt{3}}{2}$ i) ✓	13

Vraag 5 [17 punte]

No.	Antwoord/ Answer	Punte/ Marks
5.1	$(ax^3 + \frac{1}{2x})^8$ $\binom{8}{r}(ax^3)^{8-r}(\frac{1}{2x})^r$ ✓✓$=\binom{8}{r}a^{8-r}x^{24-3r}(½)^r x^{-r}$ $=\binom{8}{r}a^{8-r}x^{24-4r}(½)^r$ ✓ 24-4r=0 , r=6✓ Konstante term is/Constant term is $\binom{8}{6}\ a^2(½)^6 = 7$ $\frac{28a^2}{64} = 7$ $a^2 = 16$ a = ± 4 a = 4 (a> 0) ✓	5
5.2	1+ 9 + 25+.....+ $(2n-1)^2$ = ⅓ n$(4n^2$ -1) ✓ Toets as/Test if n=1: LK/LHS =1 RK/RHS = ⅓ (1)$(4(1)^2$-1) = 1 LK/LHS=RK/RHS✓ dus bewering is waar as/the statement is true for n=k✓ AANVAAR✓ dis waar as/ ACCEPT the statement is true for n=k; k∈ N 1 + 9 + 25 +.....+$(2k-1)^2$ = $\frac{1}{3}$ k$(4k^2$-1) ✓ Beskou/Consider n= k+1 LK/LHS = 1 + 9 + 25 + ...+ $(2k-1)^2$✓ + $(2(k+1)-1)^2$ ✓ =$\frac{1}{3}$ k ($4k^2$ -1) ✓ + $(2k+1)^2$ =$\frac{1}{3}$ k (2k+1)(2k-1) + $(2k+1)^2$ =$\frac{1}{3}$ (2k+1)(k(2k-1)+ 3(2k+1) =$\frac{1}{3}$ (2k+1)(2k^2+5k +3) =$\frac{1}{3}$ (2k+1)(2k+3)(k+1) ✓ RK/RHS=$\frac{1}{3}$ (k+1)$(4(k+1)^2$-1) ✓ =$\frac{1}{3}$(k+1)(2(k+1)+1)(2(k+1)-1) =$\frac{1}{3}$(k+1)(2k+3)(2k+1) ✓ LK/LHS = RK/RHS Die bewering is waar as n=1. As dit waar is vir n=k is dit ook waar vir n=k+1. Dus die bewering is waar $vir\ alle\ n \in N$✓ The statement is true for n=1.If it is true for n=k , it is also true for n=k+1 Thus the statement is true for all n∈ N	12

Vraag 6 [12 punte]

No.	Antwoord/ Answer	Punte/ Marks
6.1	Boog/Arc AB = $2(\frac{\pi}{3})$ =2,09✓ AB + CD =4,18m✓ AC^2 = 2^2 + 2^2-2.2.2cos$\frac{2\pi}{3}$ = 12✓ AC=$\sqrt{12}$ =3,46m✓ Omtrek/Perimeter ABDC =4,18+2(3,46) = 11,10m✓	5
6.2	Opp./Area AOB = $\frac{1}{2}(2)^2\frac{\pi}{3}$ ✓ = $\frac{2\pi}{3}$ ✓ Opp./Area AOB+COD= $\frac{4\pi}{3}$ Opp./Area ΔAOC = $\frac{1}{2}$(2)(2)sin$\frac{2\pi}{3}$ ✓ = $\sqrt{3}$ ✓ Opp./Area $\Delta AOC + \Delta BOD$ =$2\sqrt{3}$ Opp. Van venster/Area of window = $\frac{4\pi}{3}$ + $2\sqrt{3}$ a= $\frac{4}{3}$ ✓ ; b = 3 ✓ ; c =2✓	7

Vraag 7 [18 punte]

No.	Antwoord/ Answer	Punte/ Marks
7.1	$x^3 - 6x^2 + 30 = 0$ Stel/Let f(x) = x^3 - $6x^2$ + 30 f(4) = -2 < 0 ✓ f(5) = 5 > 0 ✓ Omdat f(x) <u>kontinu</u>✓ is en van teken <u>verander</u>✓ het dit 'n wortel tussen 4 en 5 f(x) is <u>continuous</u> and <u>change</u> signs so it has a root between 4 and 5. f '(x) = $3x^2 - 12x$✓ $a_{n+1} = a_n - \frac{{a_n}^3 - 6{a_n}^2 + 30}{3{a_n}^2 - 12a_n}$✓ n a_n 1 4,5 2 4,5555 3 4.5525 4 4,5525 x ≈ 4,553✓✓	8
7.2(a)	f(x) = $\begin{cases} 2x + 8 \ as \ x < -3 \\ 2 \quad as \quad x = -3 \\ x^2 - 7 \ as \ x > -3 \end{cases}$ $\lim_{x\to -3^-} f(x) = -6 + 8 = 2$ ✓ $\lim_{x\to -3^+} f(x) = 9 - 7 = 2$✓ $\lim_{x\to -3} f(x) = 2$ ✓	3
7.2(b)	f(-3) =2✓ ∴ $\lim_{x\to -3} f(x) = f(-3)$ ✓ f(x) is kontinu by x=-3/ f(x) is continuous at x=-3✓	3
7.2(c)	f ' (x) = $\begin{cases} 2 \ as \ x < -3 \\ 0 \ as \ x = -3 \\ 2x \ as \ x > -3 \end{cases}$ ✓ $\lim_{x\to -3^-} f'(x) = 2$✓ $\lim_{x\to -3^+} f'(x) = -6$✓ f(x) is nie differensieerbaar by x=-3 nie f(x) is not differentiable at x=-3✓	4

Vraag 8 [14 punte]

No.	Antwoord/ Answer	Punte/ Marks
8.1	$D_x[\frac{tan7x}{(3x^2+7)^3}] = \frac{sec^2 7x \checkmark .7 \checkmark (3x^2+7)^3 - \checkmark (tan7x).3 \checkmark (3x^2+7)^2 \checkmark .6x \checkmark}{(3x^2+7)^6 \checkmark}$	7
8.2	f(x) $= cos^4(bgtan3x)$ f'(x) $= 4 \checkmark cos^3 \checkmark (bgtan3x) \checkmark . (-\sin \checkmark (bgtan3x) \checkmark). \frac{1}{1+(3x)^2} \checkmark .3$ ✓	7

Vraag 9 [24 punte]

No.	Antwoord/ Answer	Punte/ Marks
9.1(a)	$\int (cosec^2 3x + \theta - (2x+7)^8)\, dx$ $= \frac{-cot3x}{3} \checkmark + \theta x \checkmark - \frac{(2x+7)^9}{9.2} \checkmark + k$	3
9.1(b)	$\int \frac{2}{\sqrt{9-4x^2}}\, dx$ $= 2 \int \frac{1}{\sqrt{9 \checkmark (1-\frac{4x^2}{9}) \checkmark}}\, dx = \frac{2}{3} \int \frac{1}{\sqrt{1-\left(\frac{2x}{3}\right)^2}}\, dx = \frac{2}{3} \checkmark$ bgsin $\left(\frac{2x}{3}\right) \checkmark . \frac{3}{2} \checkmark + k$	5
9.2	$V = \pi \int_0^5 \checkmark (20-4x) \checkmark\, dx$ 20-4x=0 , x=5 $= \pi[20x - \frac{4x^2 \checkmark}{2}]_0^5$ $= \pi$ [100 -50] ✓ $= 50\pi$ ✓	5
9.3	Opp./Area $= \int_0^{\frac{\pi}{12}} \checkmark sin2x \checkmark\, dx + \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \checkmark cos4x \checkmark\, dx$ $= \left[\frac{-cos2x}{2}\right] \checkmark_0^{\frac{\pi}{12}} + \left[\frac{sin4x}{4}\right] \checkmark_{\frac{\pi}{12}}^{\frac{\pi}{8}}$ $= ½[-\cos\frac{\pi}{6} + \cos 0] \checkmark + ¼[\sin\frac{\pi}{2} - \sin\frac{\pi}{3}] \checkmark$ $= ½[\frac{-\sqrt{3}}{2} + 1] + ¼[1 - \frac{\sqrt{3}}{2}] = 0{,}10 \checkmark$	11

Vraag 10 [20 punte]

No.	Antwoord/ Answer	Punte/ Marks
10.1	AB =(4-3;2+1;1-2) = (1;3;-1) of/or i +3j –k✓ DC = (-3-2;1-1;1-3) =(-5;0;-2) of/or -5i-2k✓	2
10.2	|AB| = $\sqrt{1+9+1} = \sqrt{11}$ ✓ |DC| = $\sqrt{25+4} = \sqrt{29}$✓	2
10.3	AB.DC =-5+2 =-3✓	1
10.4	-3 =$\sqrt{11}\sqrt{29}\cos\theta$ ✓ $\cos\theta$ = -0,1679677 θ =1,74 radiale/radians✓	2
10.5	AxB✓ =$\begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 4 & 2 & 1 \end{vmatrix}$✓ = i$\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}$ – j$\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ + k$\begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix}$ = i (-1-4) ✓ – j (3 – 8) ✓ + k (6+4) ✓ = -5i ✓+ 5j ✓+ 10k✓	8
10.6	$\lvert A \times B \rvert$✓= $\sqrt{25+25+100}$ = 12,25✓	2
10.7	|D| = $\sqrt{4+1+9} = \sqrt{14}$✓ γ = bgcos/arcos ✓ ($\frac{3}{\sqrt{14}}$) =0,64 radiale/radians ✓	3