α-MATHEMATICS

Alpha Mathematics

Final Examination

Eksamen 2020 **Reading time: 10 min**

Grade 11 **Time: 2½ hours**

Examiner: AM Muller **Total: 165 marks**

Moderator: R Grobler

INSTRUCTIONS AND INFORMATION

Read the instructions carefully before answering the questions:

1. This question paper consists of 7 pages, a formula sheet and an answering sheet.
2. Answer all 10 questions.
3. Non-programmable calculators may be used.
4. Questions and answers must have the same numbers.
5. Unless otherwise indicated, all answers, where necessary, must be given correct to **two decimal figures**.
6. **ALL necessary calculations must be shown clearly, except in Question 1**. No marks will be awarded if only the answer is given.
7. The diagrams in the question paper are not necessarily drawn to scale.
8. All angles are given in radians. Answers must also be given in radians if necessary.
9. Write neatly and legible.

Question 1 [20 marks]

This question must be answered on the answer page.

Each question has ONLY one correct answer and counts two (2) points. Mark the correct answer with an X on the answer page.

1.1 Solve for x : $\frac{-2}{|4-2x|} + 4 \geq 0$

(A) $x \leq \frac{7}{4}$

(B) $\frac{7}{4} \leq x \leq \frac{9}{4}; x \neq 2$

(C) $\frac{-9}{4} \leq x \leq \frac{-7}{4}; x \neq 2$

(D) $x \leq \frac{7}{4}$ or $x \geq \frac{9}{4}$

1.2 The X-axis intercept of $y = \arctan 2x - \frac{\pi}{4}$ is, $x =$ …

(A) $\frac{\pi}{4}$

(B) $\frac{-\pi}{4}$

(C) ½

(D) 2

1.3 $x^2(3x + 4y)^{10}$ has … terms

(A) 9

(B) 10

(C) 11

(D) Infinity

1.4 $(2 + 5x)^{-3}$ will converge if …

(A) $|x| < \frac{2}{5}$

(B) $|x| < 5$

(C) $|x| < \frac{1}{5}$

(D) $|x| > \frac{2}{5}$

1.5 The inverse of $f(x) = 3\arcsin\left(x + \frac{1}{2}\right) - 5$ is

(A) $f^{-1}(x) = \sin\left(\frac{x}{3} + 5\right) + \frac{1}{2}$

(B) $f^{-1}(x) = \sin\left(\frac{x+5}{3}\right) - \frac{1}{2}$

(C) $f^{-1}(x) = \sin\left(\frac{x}{3}\right) + \frac{9}{2}$

(D) $f^{-1}(x) = \sin\left(\frac{-5x}{3}\right) - \frac{1}{2}$

1.6 To decompose $\frac{x^2-4x+7}{x^2(x-1)^2(x^2+1)}$ into partial fractions we will use …

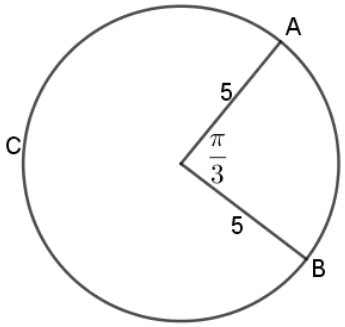
(A) $\frac{Ax+B}{x^2} + \frac{Cx+D}{(x-1)^2} + \frac{Ex+F}{x^2+1}$

(B) $\frac{A}{x^2} + \frac{B}{(x-1)^2} + \frac{C}{x^2+1}$

(C) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x-1)^2} + \frac{Ex+F}{x^2+1}$

(D) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{Ex+F}{x^2+1}$

1.7 The length of the arc ACB is



(A) 26,18 cm

(B) 5,48 cm

(C) 5,24 cm

(D) 130,90 cm

1.8 The point product of **A** $= (3; 2; a)$ and **B** $= (5; -1; 2)$, **A** • **B** $= 15$, then $a =$ …

(A) 1

(B) ± 1

(C) -1

(D) 4½

1.9 $f(x) = \begin{cases} x^2 - a \text{ if } x \leq 3 \\ 2x + 2 \text{ if } x > 3 \end{cases}$

$f(x)$ is continuous at $x = 3$, then $a =$ …

(A) ± 1

(B) 1

(C) $\sqrt{11}$

(D) $\pm\sqrt{11}$

1.10 The inverse of $f(x) = 3\cos(2x - 3)$ will be defined if the domain of f is restricted to $x \in$ ...

(A) $[0; \pi]$

(B) $[1; 2]$

(C) $x \in \mathbb{R}$; $x \neq \frac{3}{2}$

(D) $\left[\frac{3}{2}\ ;\ 3{,}07\right]$

Question 2 [11 marks]

2.1 Solve for x: $|x - 1| = \frac{x-5}{3}$ (7)

2.2 Given $y = -2|x - 3| + 4$

Determine:

(a) the salient point

(b) the Y-axis intercept

(c) the X-axis intercept(s) (4)

Question 3 [16 marks]

3.1 Given $f(x) = 1 - x^2$ and $g(x) = \sin(\pi - 3x)$. Determine $(f \circ g)(x)$ in its simplest form. (3)

3.2 Given $f(x) = \frac{3x+5}{(1-x)(1+3x)}$

(a) Decompose $f(x)$ into partial fractions. (5)

(b) Use the Power series and your answer in (a) and expand $f(x)$ in ascending powers of x up to the term that contains x^2. (8)

Question 4 [13 marks]

Factorize $f(x)$ fully in $\mathbb{C}[x]$ if $-1+2\sqrt{3}$ is a zero of

$f(x) = x^5 + 5x^4 - 2x^3 - 25x^2 - 29x - 22$ (13)

Question 5 [17 marks]

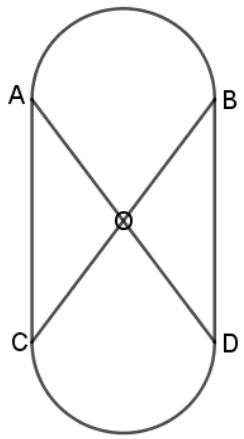
5.1 The constant term in the expansion of $\left(ax^3 + \frac{1}{2x}\right)^8$ is 7.

Determine the value of a if $a > 0$. (5)

5.2 Use Mathematical Induction and prove that the following statement applies to all $n \in \mathbb{N}$. $\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(4n^2 - 1)$ (12)

Question 6 [12 marks]

The sketch shows the design of a window. AOB and COD are sectors of a circle with midpoint O and radius 2 meters. $A\widehat{O}B = C\widehat{O}D = \frac{\pi}{3}$ radians.



6.1 Determine the perimeter ABDC of the window correct to two decimal figures. (5)

6.2 The area of the window is $\left(a\pi + c\sqrt{b}\right)$ m^2.

Determine the values of a, b and c. (7)

Question 7 [18 marks]

7.1 Prove that the equation $x^3 = 6x^2 - 30$ has a solution between 4 and 5. Use Newton's method to determine this solution correct to 3 decimal figures. (8)

7.2 Given $f(x) = \begin{cases} 2x + 8 \text{ if } x < -3 \\ 2 \quad \text{if } x = -3 \\ x^2 - 7 \text{ if } x > -3 \end{cases}$

(a) Determine the $\lim_{x \to -3} f(x)$ if it exists.

(b) Is $f(x)$ continuous at $x = -3$? Motivate.

(c) Is $f(x)$ differentiable at $x = -3$? Motivate. (10)

Question 8 [14 marks]

Differentiate the following. It is not necessary to simplify your answers.

8.1 $D_x\left[\frac{\tan 7x}{(3x^2+7)^3}\right]$ (7)

8.2 Determine $f'(x)$ if $f(x) = \cos^4(\arctan 3x)$ (7)

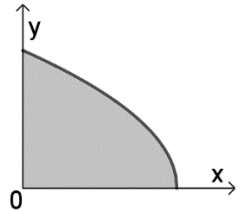
Question 9 [24 marks]

9.1 Determine the following integrals:

(a) $\int (\text{cosec}^2(3x) + \theta - (2x+8)^8)\, dx$ (3)

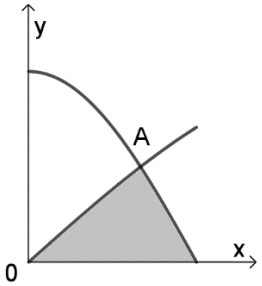
(b) $\int \frac{2}{\sqrt{9-4x^2}}\, dx$ (5)

9.2 Determine the volume of the solid when $f(x) = \sqrt{20 - 4x}$ rotates about the X-axis. Give you answer in terms of π. (5)



9.3 The graphs of $f(x) = \cos 4x$ and $g(x) = \sin 2x$ for the interval $\left[0; \frac{\pi}{8}\right]$ are shown in the sketch below. The x-coordinate of A is $\frac{\pi}{12}$.

Determine the area of the shaded part. (11)



Question 10 [20 marks]

The following vectors are given:

$$\mathbf{A} = 3i - j + 2k$$

$$\mathbf{B} = 4i + 2j + k$$

$$\mathbf{C} = -3i + j + k$$

$$\mathbf{D} = 2i + j + 3k$$

10.1 Determine the vectors **AB** and **DC**. (2)

10.2 Determine the magnitudes of **AB** and **DC**. (2)

10.3 Determine the point product **AB • DC**. (1)

10.4 Determine the size of the angle between **AB** and **DC**. (2)

10.5 Determine a vector perpendicular to **A** and **B.** (8)

10.6 Determine the area of the parallelogram formed by **A** and **B**. (2)

10.7 Determine the size of the angle between **D** and the Z-axis. (3)

- END OF THE QUESTION PAPER -