α-MATHEMATICS

Grade 10 Alpha Mathematics

Final examination 2020

Examiner: L Liebenberg **Time: 2 hours**

Moderator: R Grobler **Total: 130**

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 7 pages, a formula sheet of 1 page and a diagram sheet of 2 pages.
2. Answer ALL 9 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places.
6. Cleary show all calculations, diagrams, graphs etcetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians where necessary.
10. Write neatly and legibly.

Question 1 [20 marks]

This question must be answered **on the answer sheet**.
Every question has **ONLY** one correct answer and is worth two (2) marks.
Mark the correct answer with an **X** on the answer sheet.

1.1 The following is an example of a real number:

A $\sqrt{-1}$

B i

C i^{20}

D None of the above.

1.2 Which one of the statements below is true?

A $90°$ is the same as $\frac{\pi}{4}$ radians.

B 1 radian is the same as $60°$.

C Angles can only be measured in degrees.

D If $tan\theta = 1$ then $\theta = \frac{\pi}{4}$.

1.3 Given the polynomial $f(x) = x^3 + x^2 - 8x - 12$ and that $f(2) = -16$; $f(-2) = 0$; $f(-3) = -6$ and $f(3) = 0$.
Two **factors** of this polynomial are:

A $x = 3$ and $x = -2$

B $(x + 3)$ and $(x - 2)$

C $(x + 2)$ and $(x - 3)$

D $x = -3$ and $x = 2$

1.4 Given that $f(x) = 2x + \frac{\sqrt{x}}{3x}$ and $g(x) = (x-1)^2$.
The simplified value of $F(x) = (f \circ g)(x) =$

A $2(x-1)^2 + \frac{1}{3x-3}$

B $\left(2x + \frac{\sqrt{x}}{3x} - 1\right)^2$

C $2(x-1)^2 + \frac{\sqrt{x-1}}{3x-3}$

D $2(x-1)^2 + \frac{1}{3}$

1.5 Given that $g(x) = \frac{1}{\sqrt{x}}$

A $g'(x) = \sqrt{x}$

B $g'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

C $g'(x) = -\left(\sqrt{x}\right)^{-1}$

D $g'(x) = \frac{1}{2}x^{-2}$

1.6 $\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}-6\\5\\-4\end{bmatrix} =$

A Not possible.

B $[-8]$

C $[-6 \quad 10 \quad -12]$

D $\begin{bmatrix}-6\\10\\-12\end{bmatrix}$

1.7 Given two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ if $\boldsymbol{a}.\boldsymbol{b} = 0$, it means that:

A $\boldsymbol{a}$ and $\boldsymbol{b}$ are $\perp$

B $\boldsymbol{a}$ and $\boldsymbol{b}$ are $\parallel$

C $\boldsymbol{a} = \boldsymbol{b}$

D $|\boldsymbol{a}| = |\boldsymbol{b}|$

1.8 For which value of k will $k(2+i)+3i$ be only real?
Given that $k \in \mathbb{R}$.

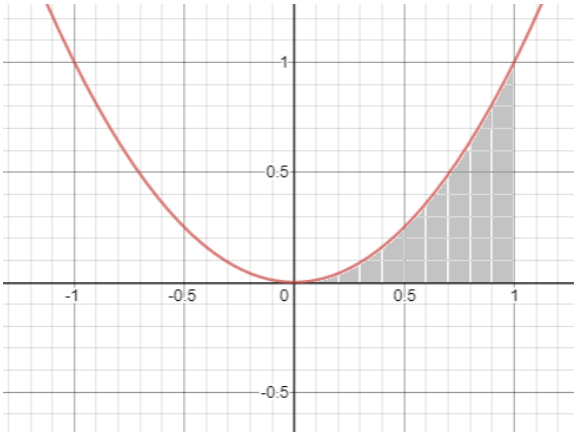
A $k = 3$

B $k = 0$

C $k = -2$

D $k = -3$

1.9 The diagram below represents the graph of $h(x) = x^2$. The area of the shaded part where $x \in [0\,;1]$ is equal to…



A $1\ unit^2$

B $\frac{1}{3}\ units^2$

C $2\ units^2$

D $\frac{1}{2}\ units^2$

1.10 $\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} =$

A 5

B $\begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$

C $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

D 3

Question 2 **[20 marks]**

2.1 Given that $z = 3 - 2i$ determine z^*, that is the conjugate of z. (1)

2.2 Simplify each expression below and leave the answer in the form $a + bi$ where $a, b \in \mathbb{R}$:

2.2.1 $i^{25}(6 - 3i)$ (2)

2.2.2 $(1 - i)^3 + (2i + 2)$ (4)

2.3 Prove that $\frac{4-2i}{1-i} = \frac{i^6}{3}(-3i - 9)$ (5)

2.4 Determine the value of a, such that $(a + 3i)(1 - 4i) = 14 - 5i$ (4)

2.5 Use DIAGRAM SHEET 1 to represent $(5 + i) + (-6i - 3)$ graphically on the set of axes. (4)

Question 3 **[10 marks]**

Decompose $\frac{5x-2}{(x-1)(x^2-4x+4)}$ into partial fractions.

Question 4 **[14 marks]**

4.1 Given that $g(x) = x^2 - 3$ and $F(x) = (f \circ g)(x) = \frac{1}{\sqrt{x^2-3}} + (x^2 - 3)$ determine:

4.1.1 $f(x)$ (2)

4.1.2 $F(2)$ (2)

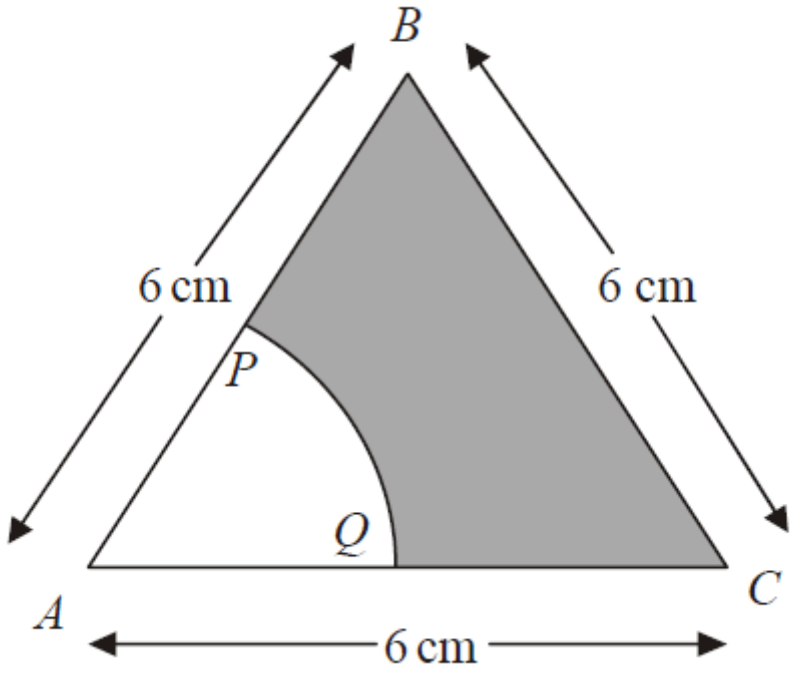
4.1.3 $g'(3)$ (2)

4.2 Sketch the following piecewise function on DIAGRAM SHEET 2 for all $x \in \mathbb{R}$. Clearly indicate all end points and intercepts with the axis.

$$f(x) = \begin{cases} x + 1 & if \quad x < -1 \\ 2 & if \quad -1 \leq x < 0 \\ x^2 & if \quad 0 \leq x < 2 \\ x & if \quad x \geq 2 \end{cases}$$ (8)

Question 5 **[9 marks]**

The diagram shows an equilateral triangle ABC with sides of length 6cm.
P is the midpoint of AB.
Q is the midpoint of AC.
APQ is a sector of a circle, centre A.
Calculate the area of the shaded region.



Question 6 **[20 marks]**

6.1 Do the following matrix calculations:

6.1.1 $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} 4 & -4 \\ 3 & 0 \\ 1 & -1 \end{pmatrix}$ (4)

6.1.2 $\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ (3)

6.1.3 A^T if $A = \begin{pmatrix} 10 & 12 & 11 \\ 13 & 15 & 14 \end{pmatrix}$ (2)

6.2 Given that

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 2 & -1 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ -4 \end{bmatrix}$$

6.2.1 Write down the 3 equations that form the matrix. (3)

6.2.2 Determine the y value by making use of Cramer's rule. Clearly indicate which matrix you are working with and how you determine the determinant. (8)

Question 7 **[15 marks]**

7.1 Given the vectors $\boldsymbol{u} = (2\,;-1)$ and $\boldsymbol{v} = (a\,;8)$. Determine the value of a, for which the vectors are at right angles to each other. (4)

7.2 Calculate the horizontal and vertical components of a vector with a magnitude of 20 and in a direction $\frac{\pi}{2}$ North of East. (2)

7.3 Determine the angle θ (in radians) between the vectors $\boldsymbol{u} = (4\,;3)$ and $\boldsymbol{v} = (5\,;\,-3)$. (6)

7.4 Determine the unit vector of $\boldsymbol{u} = (3\,;4)$. (3)

Question 8 **[6 marks]**

Solve $2x^4 + x^3 - 9x^2 - 4x + 4 = 0$ given that $x^2 - x - 2$ is a factor.

Question 9 **[16 marks]**

9.1 The volume of a revolution that exists when the graph of $y = x + q$ rotates about the x-axis between $x = 0$ and $x = 3$, is 39π. Determine the value of q, show all calculations. (10)

9.2 Determine the following (you do not have to simplify):

9.2.1 $f'(x)$ if $f(x) = 2\sqrt[4]{x} - \frac{3}{x^3} - 2$ (3)

9.2.2 $\int (5x+1)^5 dx$ (3)

END OF PAPER

ALPHA MATHEMATICS FORMULA SHEET

MATRICES AND VECTORS:

$$\text{Cramer's rule: } x_i = \frac{|A_i|}{|A|}$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y$$

CALCULUS:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

TRIGONOMETRY:

In a sector: $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$

TABLE WITH DERIVATIVES:

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)].g'(x)$

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Final examination 2020 answer sheet

Name and Surname:

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Question Total	1 [20]	2 [20]	3 [10]	4 [14]	5 [9]	6 [20]	7 [15]	8 [6]	9 [16]	TOTAL 130
Learner mark										

Question1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 [Question 2.5]

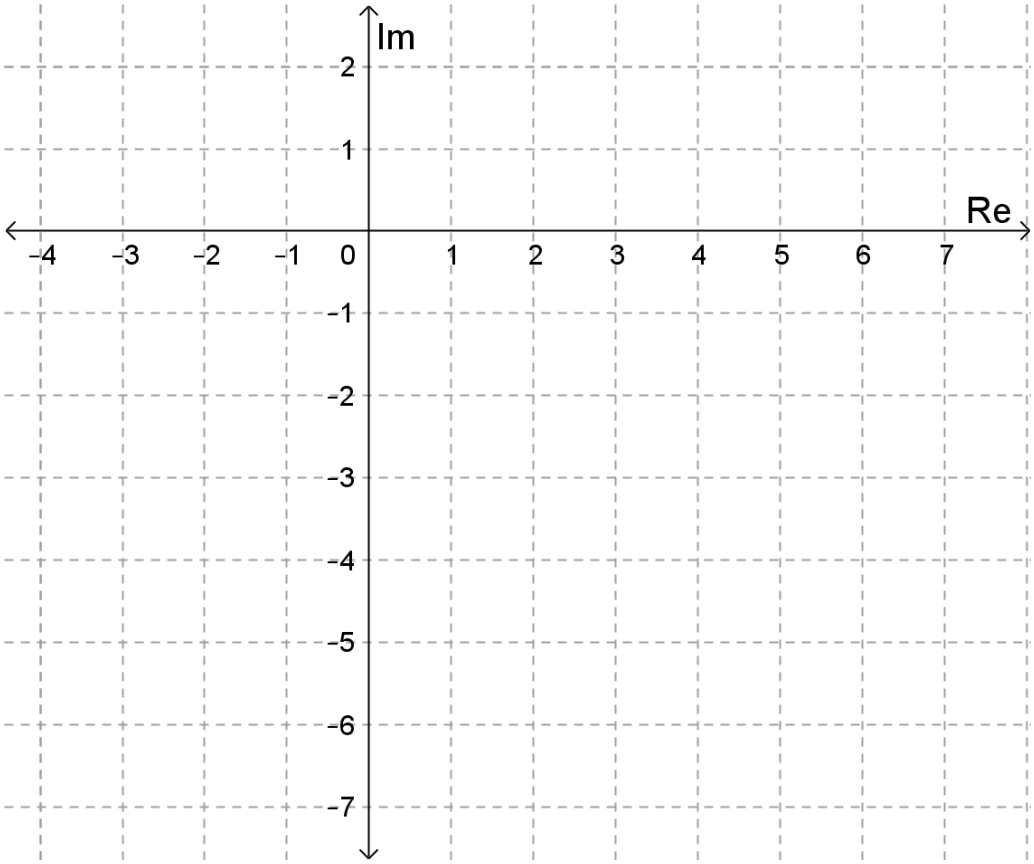

DIAGRAM SHEET 2 [Question 4.2]