α-WISKUNDE

Alpha Wiskunde Graad 12 / *Alpha Mathematics Grade 12*

Halfjaar eksamen 2020 / *Half year examination 2020*

MEMORANDUM

Totaal / *Total*: 190 punte / *marks*

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Hierdie memorandum bestaan uit 8 **bladsye. /**
This memorandum consists of 8 ***pages.***

Vraag / *Question* 1 **[20 punte / *marks*]**

NR. NO	ANTWOORD *ANSWER*	BEREKENINGE (nie vir nasien doeleindes nie) *CALCULATIONS (not for marking purpose)*	PUNTE *MARKS*
1.1	D	$y = e^x$ Helling $y' = e^x = y$ Loodlyn se helling dus: $-\frac{1}{y}$	**2**
1.2	B	$f(x) = x^2 + 2x - 15$ $f'(x) = 2x + 2 \leq 0$ $x \leq -1$	**2**
1.3	A	$f(x) = \log_3 e^x$ $f'(x) = \frac{e^x}{e^x . \ln 3} = \frac{1}{\ln 3}$	**2**
1.4	A	$y = 3e^{2x}$ $\frac{x}{3} = e^{2y}$ $y = \frac{1}{2}\ln\left(\frac{x}{3}\right)$	**2**
1.5	B	$f'(x) = 3x^2 + 2xa + b$ $f''(x) = 6x + 2a = 0$ $6(1) + 2a = 0$ $a = -3$ $1 = 1^3 - 3 + b + 1$ $b = 2$	**2**
1.6	D	$\int_1^5 \frac{1}{x}\, dx$ $= ln5 - ln1 = ln5$	**2**
1.7	C	$y = \lim_{x \to 0} \frac{x^3\left(\frac{1}{x} + \frac{5}{x^2}\right)}{x^3\left(3 - \frac{2}{x} + \frac{1}{x^3}\right)}$ $y = 0$	
1.8	D	As $x = 1$: $\ln f(1) = e\ln(1)$ $f(1) = e^0 = 1$ $\ln f(x) = e^x \ln x$ $\frac{1}{f(x)} f'(x) = e^x \ln x + e^x \frac{1}{x}$ $\frac{1}{f(1)} f'(1) = e\ln 1 + e$ Maar $f(1) = 1$ $f'(1) = e$	**2**
1.9	A	$f(x) = e^{2x}$ $f'(x) = 2e^{2x}$ $f'(0) = 2e^0 = 2$ en $f(0) = e^0 = 1$ $y - 1 = 2x - 0$ $y = 2x + 1$	**2**
1.10	B	$g'(x) = 0$ by $x = 1$ Dus draaipunt by $x = 1$ $g''(x) > 0$ dus konkaaf op	**2**

Vraag / *Question* 2 **[29 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.1 (a)	$6500 = 230e^{28k}$ $\frac{6500}{230} = e^{28k}$ $\ln\left\|\frac{6500}{230}\right\| = 28k$ $\frac{\ln\left\|\frac{6500}{230}\right\|}{28} = k$ $k = 0{,}1193 \approx 0{,}12$	✔ Verang ✔ Vereenvoudig ✔ ln beide kant ✔ Antwoord **[4 punte / *marks*]**
2.1 (b)	$N(50) = 230e^{(0,12)50}$ $= 92\,788{,}62$ Aanvaar 92788 of 92789	✔ Vervang ✔ Antwoord **[2 punte / *marks*]**
2.1 (c)	$N(t) = 230e^{0,12t}$ $N'(t) = (0{,}12)(230)e^{0,12t}$ $N'(10) = (0{,}12)(230)e^{0,12(10)}$ $N'^{(10)} = 91{,}635 \approx 91{,}64$ mense per dag (91 of 92 reg)	✔ $N'(t)$ ✔ Vervang 10 ✔ Antwoorde **[3 punte / *marks*]**
2.1 (d)	$N(t) = 230(2^{0,17t})$ $N'(t) = 230(2^{0,17t})ln2(0{,}17)$ $N'(10) = 88{,}05$ mense per dag (88 of 89 reg)	✔ $N'\ ln2$ ✔ (0,25) ✔ Anwoord **[3 punte / *marks*]**
2.2 (a)	$e^{1-x} = \frac{5}{e^{2x}}$ $e^{2x}e^{1-x} = 5$ $e^{x+1} = 5$ $x + 1 = ln5$ $x = ln5 - 1$ $x = 0{,}609 \approx 0{,}61$	✔ Vermenigvuldig e^{2x} ✔ $tel\ eksponente\ op$ ✔ ln beide kante ✔ Antwoord **[4 punte / *marks*]**
2.2 (b)	$\ln\left(\frac{1}{e^3}\right) = e^{\ln(1-2x)}$ $\ln\left(\frac{1}{e^3}\right) = 1 - 2x$ $e^{-3} = e^{1-2x}$ $-3 = 1 - 2x$ $2x = 4$ $x = 2$	✔ $1 - 2x$ ✔ e^{1-2x} ✔ Antwoord **[3 punte / *marks*]**
2.3 (a)	$y + \frac{e}{2} = \ln(2x + 1)$ $e^{y+\frac{2}{e}} = 2x + 1$ $\frac{e^{y+\frac{2}{e}} - 1}{2} = x$ $f^{-1}(x) = \frac{e^{x+\frac{2}{e}} - 1}{2}$	✔ $e^{y-\frac{2}{e}}$ ✔ +1 ✔ $\frac{e^{y+\frac{2}{e}+1}}{2}$ ✔ Antwoord **[4 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
2.3(b)	x-afsnit $(y = 0)$: $\ln(2x+1) - \frac{e}{2} = 0$ $x = \frac{e^{(0)+\frac{2}{e}}-1}{2} = \frac{e^{\frac{2}{e}}-1}{2} \approx 1{,}45$ y-afsnit $(x = 0)$: $= \ln(2(0)+1) - \frac{e}{2}$ $y = \ln(1) - \frac{e}{2} = -\frac{e}{2} \approx -1{,}36$	✔ Vervang y=0 ✔ Antwoord ✔ Vervang x=0 ✔ Antwoord **[4 punte / *marks*]**
2.3(c)	$(2x+1) > 0$ $2x > -1$ $x > -\frac{1}{2}$ Vergelyking: $x = -\frac{1}{2}$ (Net ant vol punte)	✔ $2x+1>0$ ✔ $x = -\frac{1}{2}$ **[2 punte / *marks*]**

Vraag / *Question* 3 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
3.1 (a)	$i-1 = -1+i$ $\theta = bgtan\left(-\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ (2de kwadrant) $r = \sqrt{1^2+(-1)^2} = \sqrt{2}$ $i-1 = \sqrt{2}cis\left(\frac{3\pi}{4}\right)$	✔ $\boldsymbol{i-1=-1+i}$ ✔ θ ✔ $\boldsymbol{r}$ ✔ **Antwoord** **[4 punte / *marks*]**
3.1 (b)	$(i-1)^3 = \left(\sqrt{2}cis\left(\frac{3\pi}{4}\right)\right)^3 = \left(\sqrt{2}\right)^3 cis\left(3(\frac{3\pi}{4})\right) = 2\sqrt{2}\ cis\left(\frac{9\pi}{4}\right)$ $= 2\sqrt{2}\ cis\left(\frac{\pi}{4}\right)$	✔ $\left(\sqrt{2}\right)^3$ ✔ $\frac{9\pi}{4}$ ✔ $\frac{\pi}{4}$ **[3 punte / m*arks*]**
3.1 (c)	$(\cos 3\theta + \mathrm{i}\sin 3\theta) = (cos\theta + isin\theta)^3$ $(cos\theta + isin\theta)^3$ $= \cos^3\theta + 3\cos^2\theta\, isin\theta + 3\cos\theta\, i^2 sin^2\theta + i^3 sin^3\theta$ $= \cos^3\theta + 3\cos^2\theta\, isin\theta - 3\cos\theta\, sin^2\theta - isin^3\theta$ $\therefore \sin 3\theta = 3\cos^2\theta\, sin\theta - sin^3\theta$	✔ $\left(\sqrt{2}cos\theta + \sqrt{2}isin\theta\right)^3$ ✔ ✓ binomiaal ✔ ✓Vereenvoudig ✔ Antwoord **[6 punte / *marks*]**
3.2 (a)	$2\left(\sqrt{3}-i\right)$ $\theta = bgtan\left(-\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$ 4^{de} kwadrant $r = \sqrt{\left(\sqrt{3}\right)^2+(-1)^2} = 2$ $2\left(\sqrt{3}-i\right) = 2\left(2e^{\frac{-\pi}{6}}\right) = 4e^{\frac{-\pi}{6}i}$	✔ θ ✔ $\boldsymbol{r}$ ✔ $2e^{-\frac{\pi}{6}}$ ✔ $4e^{-\frac{\pi}{6}i}$ **[4 punte / *marks*]**
3.2 (b)	$\frac{\sqrt{2}(i-1)}{\sqrt{2\left(\sqrt{3}-i\right)}} = \frac{2cis\left(\frac{3\pi}{4}\right)}{\left(4\,e^{\frac{-\pi}{6}i}\right)^{\frac{1}{2}}} = \frac{2e^{\left(\frac{3\pi}{4}\right)i}}{2\,e^{\frac{-\pi}{12}i}} = e^{\frac{5\pi}{6}i}$ $= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$	✔ Vervang 3.1(a) en 3.2(a) ✔ ✓$\sqrt{2}e^{-\frac{\pi}{12i}}$ ✔ $\frac{7\pi}{4} + \frac{\pi}{12}i$ ✔ $e^{\frac{11\pi}{6}i}$ ✔ ✓$-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ **[7 punte / *marks*]**

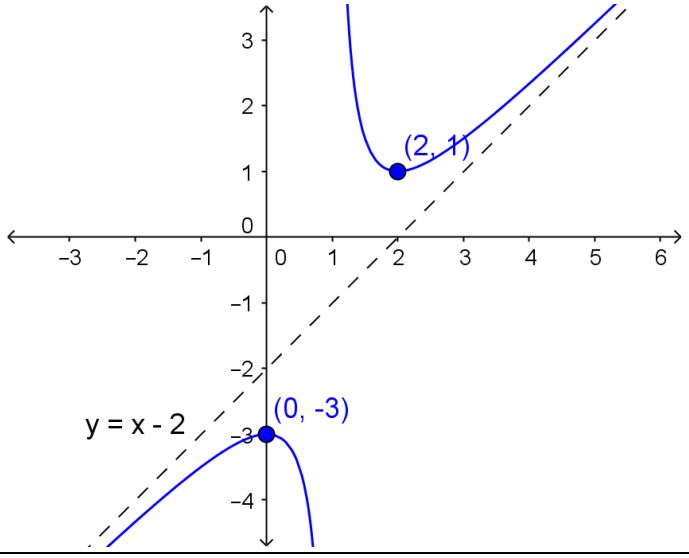
Vraag / *Question* 4 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
4.1 (a)	$\cos y y' = \frac{e^x x - e^x}{x^2} - \frac{(y^2 + x(2y)y')}{x\,y^2}$ $xy^2\cos y y' = \frac{xy^2(e^x x - e^x)}{x^2} - y^2 - 2xyy'$ $xy^2\cos y y' + 2xyy' = \frac{y^2(e^x x - e^x)}{x} - y^2$ $y'(xy^2\cos y + 2xy) = \frac{y^2(e^x x - e^x) - xy^2}{x}$ $y' = \frac{y^2(e^x x - e^x) - xy^2}{x\,(xy^2\cos y + 2xy)}$ $y' = \frac{ye^x\,(x - 1) - xy}{x^2\,(2 + y\cos(y))}$ OF $lnxy^2 = lnx + 2lny, \frac{dy}{dx} = \frac{1}{x} + \frac{2}{y}.y'$ **Dus** $\cos y\,.y' = \frac{e^x x - e^x}{x^2} - \frac{1}{x} - \frac{2}{y}.y'$	✔ $\cos y y'$ ✔ ✓✓ $\frac{e^x x - e^x}{x^2}$ ✔ ✓✓$y^2 + x(2y)y'$ ✔ $x\,y^2$ ✔ ✓Vereenvoudig ✔ Gemene faktor teller en noemer (y) **[11 punte / *marks*]**
4.1 (b)	$y' = \frac{ye^x\,(x - 1) - xy}{x^2\,(2 + y\cos(y))}$ $y'_{(1;\frac{15}{e})} = \frac{-\frac{15}{e}}{2 + \frac{15}{e}\cos(\frac{15}{e})} = -0{,}92$	✔ $ye^x\,(x - 1) = 0$ ✔ Antwoord Of net antwoord 2 punte **[2 punte / *marks*]**
4.2(a)	$Dx[x^2 + y^2 - 2x + 3y = 8]$ $2x + 2yy' - 2 + 3y' = 0$ $y'(2y + 3) = 2 - 2x$ $y' = \frac{2 - 2x}{2y + 3}$	✓✓✓✓$2x + 2yy' - 2 + 3y'$ ✔ Antwoord **[5 punte / *marks*]**
4.2(b)	$y'' = \frac{-2(2y + 3) - 2y'(2 - 2x)}{(2y + 3)^2}$ $y'_{(-2;0)} = \frac{2 - 2(-2)}{2(0) + 3} = \frac{6}{3} = 2$ $y''_{(-2;0)} = \frac{-2(3) - 2(2)(2 - 2(-2))}{(3)^2} = \frac{-6 - 24}{9} = -\frac{10}{3}$	✔ ✓$-2(2y + 3) - 2y'(2 - 2x)$ ✔ $(2y + 3)^2$ ✔ $y' = 2$ ✔ y'':vervang y' waarde ✔ Antwoord **[6 punte / *marks*]**

Vraag / *Question* 5 **[23 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
5.1	$f(x) = \frac{(ax\ +\ 3)(x+\ b)}{(x\ +\ c)(x+d)}$ $b = d$ sal punt wees waar funksie nie geldig $x + b = 2$, dus $x - 2$ en $b = d = -2$ $x + c$ is die vertikale assimptoot $x = -1$, dus $c = 1$ Horisontale assimptoot is waar $\lim_{x\to\infty} \frac{ax^2+3x+abx+3b}{x^2+bx+cx+cd}$ $\lim_{x\to\infty} \frac{x^2(a+\cdots)}{x^2(1+\cdots)} = 2$, dus $\frac{a}{1} = 2$, $a = 2$	✔ ✓ $b = d = -2$ ✔ ✓$c = 1$ ✔ ✓$a = 2$ [6 punte / *marks*]
5.2	Indien $d = -1$, dan is $f(x) = \frac{(2x\ +\ 3)(x-2)}{(x\ +\ 1)(x-1)}$ Vertikale assimptote is dus $x = -1$, $x = 1$ en horisontale assimptoot is $y = 2$	✔ ✓✓ assimptote [3 punte / *marks*]
5.3	$f(x) = \frac{(x\ +\ 3)(x-1)}{(x\ )(x)} = \frac{x^2+2x-3}{x^2}$ $f'(x) = \frac{(2x+2)(x^2)-(2x)(\ x^2+2x-3)}{(x^2)^2} = 0$ $2x^3 + 2x^2 - 2x^3 - 4x^2 + 6x = 0$ $-2x^2 + 6x = 0$ $x(x-3) = 0$ $x = 0$ of $x = 3$ $x = 0$ is assimtoot en dus nie draaipunt nie Draaipunt $x = 3$: $f(3) = \frac{3^2+2(3)-3}{3^2} = \frac{12}{9}$ $\left(3;\frac{4}{3}\right)$	✓✓ $f'(x)$ ✓$-2x^2 + 6x$ ✓ $x(x-3)$ ✓✓ $x = 0$ of $x = 3$ ✓✓ $\left(3;\frac{4}{3}\right)$ [8 punte / *marks*]
5.4	$f'(x) = \frac{-2x^2+6x}{x^4} = \frac{6-2x}{x^3}$ Bepaal die aard van die draaipunt: $f''(x) = \frac{(-2)x^3-(6-2x)(3x^2)}{x^6}$ $= \frac{-2x^3-18x^2+6x^3}{x^6}$ $= \frac{4x^3-18x^2}{x^6} = \frac{x^2(4x-18)}{x^2(x^4)}$ $f''(3) = \frac{4(3)-18}{3^4} = \frac{-6}{3^4} < 0$ Lokale maksimum	✔ ✓✓$f''(x)$ ✓$\frac{4x-18}{x^4}$ ✓$f''(3) < 0$ ✓ Lokale maksimum [6 punte / *marks*]

Vraag / *Question* 6 **[22 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
6.1	y-afsnit $(x = 0)$: $y = \frac{(0)^2 - 3(0) + 3}{(0) - 1} = -3$ y-afsnit is -3.	✔ ✓ Antwoord **[2 punte / *marks*]**
6.2	Vertikale assimptoot: $x - 1 = 0 \rightarrow x = 1$ $\frac{x^2 - 3x + 3}{x - 1} = x - 2 + \frac{1}{x-1}$ Skuins assimptoot: $y = x - 2$ 1 | 1 -3 3 / 1 -2 / 1 -2 1	✔ Vertikale assimptoot ✔ $x = 1$ ✔ Skuins assimptoot ✔ $y = x - 2$ **[4 punte / *marks*]**
6.3	$f(x) = \frac{x^2 - 3x + 3}{x - 1}$ $f'(x) = \frac{(2x - 3)(x-1) - (x^2 - 3x + 3)(1)}{(x - 1)^2} = 0$ $2x^2 - 5x + 3 - x^2 + 3x - 3 = 0$ $x^2 - 2x = 0$ $x(x - 2) = 0$ $x = 0$ of $x = 2$ $f(0) = -3$ $f(2) = \frac{(2)^2 - 3(2) + 3}{2 - 1} = 1$ $(0; -3)$ $(2; 1)$	✔ ✓ $(2x - 3)(x - 1)$ ✔ ✓ $(x^2 - 3x + 3)(1)$ ✔ $(x - 1)^2$ ✔ $x^2 - 2x$ ✔ ✓ $x = 0$ of $x = 2$ ✔ $(0; -3)$ ✔ $(2; 1)$ **[10 punte / *marks*]**
6.4		✔ $y = x - 2$ ✔ $x = 1$ ✔ (2,1) ✔ (0,3) ✔ ✓ Vorm **[6 punte / *marks*]**

Vraag / *Question* 7 **[24 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
7.1 (a)	$\int 12x + 6\,dx$ $= 6x^2 + 6x + k$ Gradiënt by $x = 0$ is -36 $6x^2 + 6x - 36 = 0$ $x^2 + x - 6 = 0$ $(x + 3)(x - 2) = 0$ $x = -3$ of $x = 2$ $g''(x) = 12x + 6$ $g''(-3) = 12(-3) + 6 = -30 < 0$ Lokale maksimum $g''(-2) = 12(2) + 6 = 30 > 0$ Lokale minimum	✔ ✓ $6x^2 + 6x$ ✔ $x = 3$ ✔ $x = -2$ ✔ Gradiënt -36 ✔ $g''(-3) < 0$ Lokale maks ✔ $g''(2) > 0$ ✓ Lokale. min **[9 punte / *marks*]**
7.1 (b)	$g''(x) = 12x + 6 = 0$ $2x = -1$ $x = -\frac{1}{2}$ $g''(0) = 6 > 0$ $g''(-1) = -6 < 0$ Dus $x = -\frac{1}{2}$ is 'n buigpunt.	✔ $g''(x) = 0$ ✔ $x = -\frac{1}{2}$ ✔ $g''(0) > 0$ ✔ $g''(0) < 0$ ✔ Dus buigpunt **[5 punte / *marks*]**
7.1(c)	Konkaaf op: $x > -\frac{1}{2}$	✔ Konkaaf op ✔ Antwoord **[2 punte / *marks*]**
7.2(a)	$x = 0$ en $x = 2$ $x = 0$ helling negatief, dus lokale maksimum $x = 2$ helling positief, dus lokale minimum	✔ ✓ Antwoorde ✔ Lokale maksimum ✔ Lokale minimum **[4 punte / *marks*]**
7.2(b)	Lees draaipunt van $f'(x)$ van grafiek is buigpiunt $(1; -3)$	✔ ✓ $(1; -3)$ **[2 punte / *marks*]**
7.2(c)	Stygend $f'(x) > 0$ $x \leq 0$ of $x \geq 2$	✔ $x \leq 0$ ✔ $x \geq 2$ **[2 punte / *marks*]**

Vraag / *Question* 8 **[9 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*	PUNTE / *MARKS*
8.1	$f(x) = e^x + \frac{1}{e^{x-1}} - 3$ $f'(x) = e^x + e^{-x+1}(-1) = 0$ $e^x - \frac{e}{e^x} = 0$ $e^{2x} - e = 0$ $e^{2x} = e^1$ $x = \frac{1}{2}$	✓ e^x ✓ e^{-x+1}✓ (-1) ✓ $e^{2x} - e$ ✓ $x = \frac{1}{2}$ **[5 punte / *marks*]**
8.2	$f'(x) = e^x - e.e^{-x}$ $f''(x) = e^x - e.e^{-x}(-1) = e^x + \frac{e}{e^x} > 0$ Dus lokale minimum	✓ e^x ✓ $-e.e^{-x}$ ✓ (-1) ✓ min **[4 punte / *marks*]**
8.3	$Oppervlakte = \int_0^{\frac{1}{2}} (e^x + e^{-x+1} - 3)\,dx$ $= [e^x + e^{-x+1} - 3x]_0^{\frac{1}{2}}$ $= e^{\frac{1}{2}} + e^{(\frac{1}{2})} - 3\left(\frac{1}{2}\right) - (e^0 + e^{0+1} + 0)$ $= e - \frac{5}{2}$ $= 0{,}2183 \approx 0{,}22$	✓ Vervang intervalle ✓ ✓✓ Integreer ✓ Vervang grense in ✓ Vereenvoudig ✓ Antwoord **[7 punte / *marks*]**

Vraag / *Question* 9 **[8 punte / *marks*]**

NR / *NO*	ANTWOORD / *ANSWER*		
9.(a)	$\int \frac{e}{2x-1}$ $= \frac{e.\ln(2x-1)}{2} + k$	✓	✓ ✓
9.(b)	$\int \frac{e}{(2x-1)ln3}$ $= \frac{e\log_3(2x-1)}{2} + k$	✓	✓ ✓ ✓
9.(c)	$\int 5^{2x-3}\,dx$ $= \frac{5^{2x-3}}{2\,ln5} + k$	✓	✓ ✓ ✓

- EINDE VAN DIE MEMORANDUM / *END OF THE MEMORANDUM* -