

Alpha Mathematics
Grade 12 Exam Paper
October 20, 2017

Total: 200 marks

Time: 3 hours

INSTRUCTIONS

1. Answer all questions on this exam paper and hand it in. Remember to write your name and ID number on the front sheet.
2. Write neatly and legibly.
3. Non-programmable calculators may be used.
4. The diagrams on the question paper are not necessarily drawn to scale.
5. All angles are given in radians. Answers must be given in radians if necessary.
6. Unless indicated otherwise, all answers, where applicable must be given correct to two decimal digits.
7. This paper consists of 10 questions and 3 formula sheets.
8. Question 1 consists of 10 multiple choice questions. Answer it on the answer sheet. This answer sheet is in the front of the paper. Please do NOT remove it.
9. For all other questions, all necessary calculations must be shown clearly. The correct answer alone will not necessarily lead to full marks.
10. There is extra space at the end of the paper. Clearly indicate if you use this space for a question.

QUESTION 1 [20 marks]

Answer this QUESTION on the answer sheet by marking A, B, C or D with an X.

Please don't remove this page from the paper. The questions count 2 marks each.

1.1 If $f(x) = \log x$, then $f'(x) =$

(A) $\frac{1}{x}$

(B) $\frac{1}{x \cdot \ln e}$

(C) $\frac{1}{x \cdot \ln x}$

(D) $\frac{1}{x \cdot \ln 10}$

1.2 Given $f(x) = \ln(x + 1)$. Then the inverse function of f in the form $f^{-1}(x) = \dots$

(A) e^{x-1} (B) $e^x - 1$ (C) e^{x+1} (D) $e^x + 1$

1.3 If $\frac{dy}{dx} = 4x$ and $y = 4$ if $x = 0$, then $y = \dots$

(A) $4x + 2$

(B) $4 + x^2$

(C) $2x^2 + 4$

(D) $2 + 4x^2$

1.4 The graph of $y = |2x - 4| - 2$ has a salient point at ...

(A) $(-4; -2)$

(B) $(4; -2)$

(C) $(2; -2)$

(D) $(-2; -2)$

1.5 Which one of the following power series' extensions will converge if $|x| < 4$?

(A) $\frac{1}{1+4x}$

(B) $\frac{1}{4+x}$

(C) $\frac{1}{2-4x}$

(D) $\frac{1}{2x-4}$

1.6 $\sqrt{5} - 2$ is a zero of a polynomial. Which of the following will be a factor of the polynomial?

(A) $x^2 + 4x - 1$

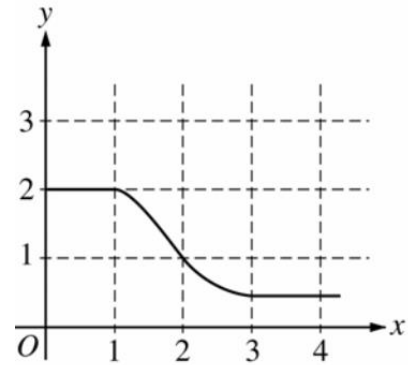
(B) $x - 2 - \sqrt{5}$

(C) $x^2 - 4x - 1$

(D) $x - 2 + \sqrt{5}$

1.7 The sketch on the right shows the graph of f . If $\int_1^3 f(x) dx = 2,3$ and $F'(x) = f(x)$, then $F(3) - F(0) = \dots$

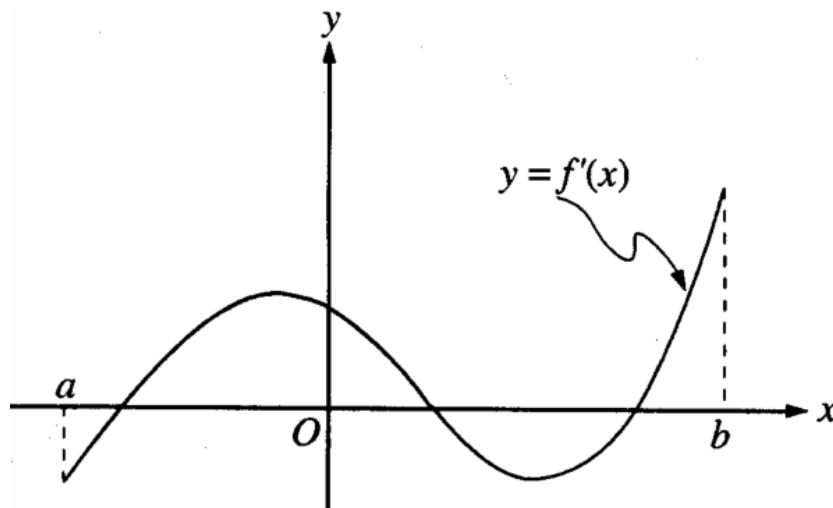
- (A) 0,3
 (B) 1,3
 (C) 3,3
 (D) 4,3



1.8 If $y = 2x - 8$, for which value of x will the product xy have a minimum value?

- (A) -16 (B) -8 (C) -4 (D) 2

1.9 The graph of f' , the derivative of f , is shown.



Which of the following describe the local stationary points of f in the interval $(a; b)$?

- (A) One local maximum and two local minimum points.
 (B) One local minimum and two local maximum points.
 (C) Three local minimum points.
 (D) Three local maximum points.

1.10 If $e^{f(x)} = 1 + x^2$ then $f'(1) = \dots$

- (A) $\frac{1}{2}$ (B) 1 (C) 4 (D) $2e^2$

Answer the following questions on the paper on the lines provided after each question.

QUESTION 2 [20 marks]

2.1 A micro biologist studies a certain bacterium. The number of bacteria in the sample can be determined by the formula $N = 5,85(2^{kt})$ where t is the time in hours, k is a constant and N is the number of bacteria in millions.

(a) Determine how many million bacteria there were initially. (2)

(b) Calculate the value of k , correct to 4 decimal digits, if there are 7,68 million bacteria after 20 hours (3)

(c) Use $k = 0,02$ and calculate the tempo at which the bacteria increase after 20 hours. (4)

3.2 Use de Moivre's theorem and rationalise the denominator of $\frac{16}{(-1-i)^8}$.

Work in polar form and use root form and π if necessary.

Give the answer in rectangular form, $a + bi$.

(6)

3.3 Use mathematical induction and prove that for all $n \in \mathbb{Z}$:

(8)

$$-1 + 4 - 9 + \dots + (-1)^n n^2 = \frac{(-1)^n n(n+1)}{2}$$

8.3 Determine the x - and y - intercepts of the graph of f . (3)

8.4 Use the information obtained and make a sketch graph of $y = f(x)$ on the set of axes given underneath, if it is also given that the graph has a horizontal asymptote at $y = 0$. (6)

