

# $\alpha$ -WISKUNDE

Alpha Wiskunde

MEMORANDUM

21 Oktober 2019  
Graad 12

Tyd: 3 ure  
Totaal: 200 punte

VRAAG 1 [20 PUNTE]

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

**Memo**

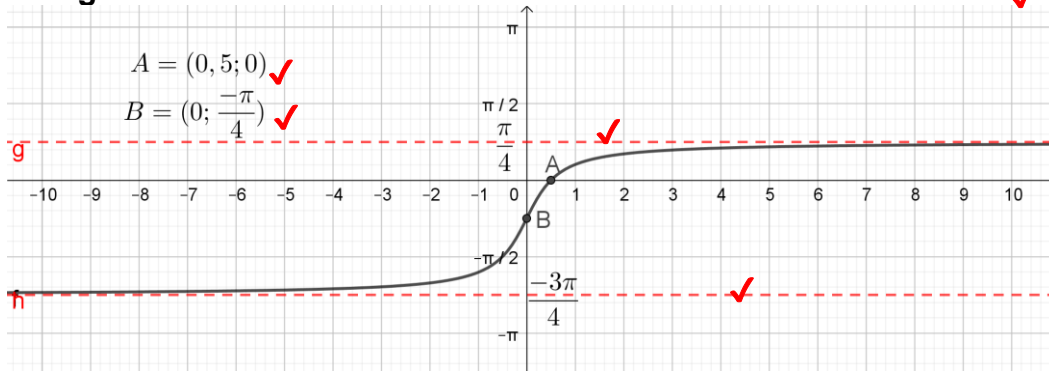
2.1 (a)	$\frac{A(1000)}{A_0} = e^{-1,21 \times \frac{1}{10000} \times 1000} \checkmark$ $= 0,89 \checkmark$	1: Vervang 1: Antwoord <b>[2]</b>
(b)	$\frac{A(t)}{A_0} = 0,75 = e^{-1,21 \times \frac{1}{10000} \times t}$ $\therefore -1,21 \times 0,0001t = \ln 0,75 \checkmark$ $t = \ln 0,75 \div \left(\frac{10\,000}{-1,21}\right) \checkmark$ $t = 2377,54 \text{jaar} \checkmark$	1: Verander na $\ln$ 1: Kry $t$ alleen 1: Antwoord <b>[3]</b>
(c)	$\frac{A(t)}{A_0} = \frac{1}{2} = e^{-1,21 \times \frac{1}{10000} \times t}$ $\therefore -1,21 \times 0,0001t = \ln \frac{1}{2} \checkmark$ $t = \ln 0,5 \div (-1,21 \times 0,0001) \checkmark$ $t = 5730 \text{jaar} \checkmark$	1: Verander na $\ln$ 1: Kry $t$ alleen 1: afronding na naaste 10 <b>[3]</b>

2.2	As $x \geq -3$ : $\checkmark$ $2x + 6 = -2x + 3 \checkmark$ $4x = -3$ $x = -\frac{3}{4}$ As $x < -3$ : $\checkmark$ $-2x - 6 = -2x + 3 \checkmark$ Geen opl $\checkmark$	1: Voorwaarde 1: Sonder abs waarde 1: Antwoord 1: Voorwaarde 1: Sonder abs waarde 1: Antwoord Maks 4 sonder voorwaardes <b>[6]</b>
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	$(x^3)^{8-r} \cdot \left(\frac{1}{x}\right)^r = 0 \checkmark$ $r = 6 \checkmark$ $\binom{8}{6} \left(\frac{x^3}{2}\right)^2 \checkmark \times \left(-\frac{2}{x}\right)^6 \checkmark$ $= 448 \checkmark$	1: Formule vir $r$ (nie nodig) 1: Waarde van $r$ 1: Vervang reg in formule (opvolg) 1: Antwoord (opvolg) <b>[5]</b>
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**Vraag 3.1**

vorm ✓



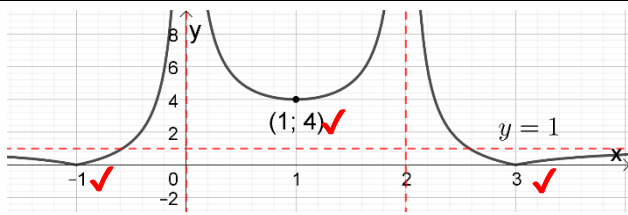
<p>3.2 (a)</p>	<p>ABDC koordevierhoek, dus <math>\hat{D} = \frac{\pi}{3}</math> ✓                  Stel <math>BD = CD = x</math>  <math>BC^2 = 5^2 + 5^2 - 2 \cdot 5 \cdot 5 \cos\left(\frac{2\pi}{3}\right) = x^2 + x^2 - 2x^2 \cos\left(\frac{\pi}{3}\right)</math> ✓  <math>x^2 = 75</math> ✓; <math>x = 5\sqrt{3}</math>                  OF                  ABCD 'n vlieër en <math>\hat{B} = 90^\circ</math> ✓                  Dus <math>\frac{BD}{AB} = \tan \frac{\pi}{3}</math> ✓; dus <math>BD = 5 \tan \frac{\pi}{3} = 5\sqrt{3}</math> ✓</p>	<p>1: Hoek D                  1: cosinus reël                  1: antwoord  <b>[3]</b></p>
<p>(b)</p>	<p>Oppervlakte ABDC = <math>2 \times \frac{1}{2} \times 5 \times 5\sqrt{3} = 25\sqrt{3}</math> ✓✓                  Oppervlakte sektor ABC = <math>\frac{1}{2} \times 25 \times \frac{2\pi}{3} = 26,18</math> ✓                  Oppervl gearseerde gedeelte = <math>25\sqrt{3} - 26,18 = 17,12</math> ✓</p>	<p>2: Oppervlakte ABCD                  1: Sektor                  1: Totale oppervl.  <b>[4]</b></p>

<p>3.3 (a)</p>	<p><math>r^2 = 3 + 1</math> ✓  <math>r = 2</math> ✓                  en <math>\theta = \pi - \frac{\pi}{6}</math> ✓  <math>= \frac{5\pi}{6}</math>  <math>2 \text{cis} \frac{5\pi}{6}</math> ✓</p>	<p>1: Bereken <math>r</math>                  1: <math>r</math>                  1: regte kwadrant                  1: Antwoord                  (<math>\frac{\pi}{6}</math> kan 1 punt kry)  <b>[4]</b></p>
<p>(b)</p>	<p><math>\frac{(2^{12} \text{cis}(10\pi))}{32 \text{cis}(\frac{\pi}{3})}</math>  <math>= 128 \text{cis}(\frac{29\pi}{3})</math> ✓</p>	<p>1: <math>2^{12}</math> en hoek                  2: 128 en hoek  <b>[3]</b></p>
<p>(c)</p>	<p><math>x = 128 \cos\left(\frac{29\pi}{3}\right) = 64</math> ✓  <math>y = 128 \sin\left(\frac{29\pi}{3}\right) = -64\sqrt{3}</math> ✓                  Dus <math>64 - 64\sqrt{3}</math></p>	<p>1: <math>x</math>                  1: <math>y</math>  <b>[2]</b></p>

4.1 (a)	Die graad van die polinoom moet onewe wees ✓ Want komplekse nulpunte kom in pare voor. ✓	1: stelling 1: rede <b>[2]</b>
(b)	$x + 1 = \pm 4i$ ✓ $x^2 + 2x + 1 = -16$ ✓ $x^2 + 2x + 17$ is 'n faktor. ✓ <p style="text-align: center;"><b>OF</b></p> $(x + 1 + 4i)(x + 1 - 4i)$ ✓ $= x^2 + 2x + 1 + 16$ ✓ $= x^2 + 2x + 17$ ✓  $P(x) = (x^2 + 2x + 17)(x^2 + 1)$ ✓ Dus geen reële nulpunte nie. ✓	3: enige metode vir faktor  2: Ander faktor. Hierdie kon 1'e stap wees, dan dadelik 5 punte. 1: Geen <b>[6]</b>
4.2	Stel $n = 1$ : LK = $\cos\theta + i\sin\theta$ en RK = $\cos\theta + i\sin\theta$ ✓ Dus waar vir $n = 1$ . Aanvaar waar vir $n = k$ : $(\cos\theta + i\sin\theta)^k = \cos(k\theta) + i\sin(k\theta)$ ✓ Stel $n = k + 1$ : $LK = (\cos\theta + i\sin\theta)^{k+1} = (\cos\theta + i\sin\theta)^k \cdot (\cos\theta + i\sin\theta)$ ✓ $= (\cos(k\theta) + i\sin(k\theta))(\cos\theta + i\sin\theta)$ ✓ $= \cos(k\theta) \cdot \cos\theta + \cos(k\theta) \cdot i\sin\theta + i\sin(k\theta) \cdot \cos\theta + i^2 \sin(k\theta) \cdot \sin\theta$ ✓ $= \cos(k\theta) \cdot \cos\theta - \sin(k\theta) \cdot \sin\theta + i(\sin(k\theta) \cdot \cos\theta + \cos(k\theta) \cdot \sin\theta)$ ✓ $= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$ ✓ $= \cos((k + 1)\theta) + i\sin((k + 1)\theta)$ ✓ = RK ✓ Dus bewering waar vir $n = 1$ . As dit waar is vir $n = k$ , dan is dit ook waar vir $n = k + 1$ . Dus waar $\forall x \in \mathbb{N}$ ✓	1: $n = 1$  1: Aanvaar $n = k$  1: eksponent reël 1: De Moivre 1: Vervang $n = k$ 1: Vermenigvuldig hakies uit. 1: $i^2 = -1$ 1: Haal $\theta$ gemeenskaplik uit. 1: Regterkant  1: Storie <b>[10]</b>

5.1	<p>Afstand = <math>(3; -1 - a; -5)</math> ✓                      Dus <math>\sqrt{9 + 1 + 2a + a^2 + 25} = 5\sqrt{2}</math> ✓  <math>a^2 + 2a + 35 - 50 = 0</math> ✓  <math>(a + 5)(a - 3) = 0</math>  <math>a = -5</math> ✓</p>	<p>1: Afstand                      1: Formule                      1: Vereenvoudig                      1: Antwoord</p> <p style="text-align: right;"><b>[4]</b></p>
5.2 (a)	<p>Lengte van vektor is <math>\sqrt{1 + 9 + 25} = \sqrt{35}</math> ✓  <math>\gamma = \text{bgcos}\left(\frac{5}{\sqrt{35}}\right)</math> ✓  <math>= 0,56</math> ✓ radiale</p>	<p>1: Lengte                      1: Formule en vervang                      1: Antwoord</p> <p style="text-align: right;"><b>[3]</b></p>
(b)	<p><math>b \times c = \begin{vmatrix} i &amp; j &amp; k \\ 2 &amp; -1 &amp; 0 \\ -3 &amp; 0 &amp; 1 \end{vmatrix}</math> ✓  <math>= i(-1) - j(2) + k(-3)</math> ✓  <math>= -i - 2j - 3k</math> ✓                      Oppervl = <math>\sqrt{1 + 4 + 9}</math> ✓  <math>= \sqrt{14}</math> ✓</p>	<p>1: Matriks                      1: Bereken matriks                      1: Vereenvoudig                      1: Formule (opvolg)                      1: antwoord</p> <p style="text-align: right;"><b>[5]</b></p>
(c)	<p><math>\cos\theta = \frac{a \cdot b}{ a  b }</math> ✓  <math>= \frac{2 \cdot (-3)}{\sqrt{35} \times \sqrt{5}}</math> ✓  <math>\cos\theta = \frac{-1}{\sqrt{175}}</math>  <math>\theta = 1,65</math> radiale ✓</p>	<p>1: Verwerk formule                      1: <math>a \cdot b</math>                      1: <math>a \times b</math>                      1: Antwoord (volg op)</p> <p style="text-align: right;"><b>[4]</b></p>
5.3	<p><math> A - kI  = \begin{vmatrix} -5 - k &amp; 2 \\ -9 &amp; 6 - k \end{vmatrix} = 0</math>  <math>(-5 - k)(6 - k) + 18 = 0</math>  <math>k^2 - k - 12 = 0</math> ✓  <math>(k - 4)(k + 3) = 0</math>  <math>k = 4</math> of <math>k = -3</math> ✓</p>	<p>1: <math>-5 - k</math>                      1: <math>6 - k</math>                      1: Determinant berekening                      1: vereenvoudig                      1: Antwoord</p> <p style="text-align: right;"><b>[5]</b></p>

6.1	Diffbaar: $a\checkmark = 2b(2) - a\checkmark$ , dus $a = 2b\checkmark$ Kontinu: $2a + 2b - 4 = 4b - 2a + a\checkmark$ $a = 2\checkmark$ en $b = 1\checkmark$	2: Elke kant diff baar 1: Vereenvoudig  1: Kontinu 2: $a$ en $b$  <b>[6]</b>
6.2 (a)	$f(x) = 2\log x \times b \tan(2x)$ $f'(x) = \frac{2\checkmark}{x \ln 10\checkmark} \cdot b \tan(2x) + \checkmark 2 \log x \cdot \frac{2\checkmark}{1+4x^2\checkmark}$ OF $f'(x) = \frac{2x\checkmark}{x^2 \ln 10\checkmark} \cdot b \tan(2x) + \checkmark \log(x^2) \cdot \frac{2\checkmark}{1+4x^2\checkmark}$	1: 2 1: $\ln 10$ 1: produkreël 1: 2 teller 1: $4x^2$  <b>[5]</b>
(b)	$-\sin\checkmark\sqrt{x} \cdot \frac{1}{2}\checkmark x^{-\frac{1}{2}\checkmark}$  OF  $\frac{-\sin\checkmark\sqrt{x}}{2\checkmark\sqrt{x}\checkmark}$	1: sin 1: $\frac{1}{2}$ 1: eksponent  <b>[3]</b>
(c)	$e^y \cdot y'\checkmark = 2^{xy} \ln(2)\checkmark \times (y\checkmark + xy'\checkmark)$ $\frac{dy}{dx} = \frac{y \cdot 2^{xy} \cdot \ln(2)\checkmark}{e^y - x \cdot 2^{xy} \cdot \ln(2)\checkmark}$	4: Elke term, soos getoon 1: Antwoord  <b>[5]</b>

7.1 (a)	$A=f''(x)$ ✓ $B=f'(x)$ ✓	1: A 1: B	[2]
(b)	$x = 0; 2; 4$ ✓ want die 2e afgeleide is nul ✓ en dit verander van teken ✓.	1: Al drie antwoorde 1: rede 1: rede	[3]
(c)	$x = 0; 4$ ✓ want daar is $f'(x) = 0$ ✓	1: Beide antwoorde 1: Rede	[2]
7.2 (a)	Horisontaal $y = -1$ ✓; Vertikaal $x = 0$ ✓ en $x = 2$ ✓	1: $y = -1$ 1: $x = 0$ 1: $x = 2$	[3]
(b)	$f'(x) = 0$ ✓ $\frac{(2x - 2)(2x - x^2) - ((x^2 - 2x - 3)(2 - 2x))}{(x^2 - 2x)^2} = 0$ $-2x^3 + 6x^2 - 4x + 2x^3 - 6x^2 - 2x + 6 = 0$ ✓ $6x = 6; x = 1.$ Dus (1 ✓; -4 ✓)	1: $f'(x) = 0$ 2: Elke term in teller 1: Vereenvoudig 1: $x = 6$ 1: $y = -4$	[6]
(c)	 <p>Asimptote ✓            Vorm ✓</p>	2: x-afsnitte 1: Draaipunt 1: Asimptote 1: Vorm	[5]

<p>8.1 (a)</p>	$\int \left( \operatorname{cosec}^2 2x - 1 + \frac{1}{2}(1 + \cos(2x)) \right) dx$ $= -\cot 2x \cdot \frac{1}{2} - x + \frac{1}{2} \left( x + \frac{1}{2} \sin(2x) \right) + k$ <p style="text-align: center;">OF</p> $= \frac{-\cot 2x}{2} - \frac{x}{2} + \frac{\sin 2x}{4} + k$	<p>1: Identiteit <math>\cot^2 x</math>            1: Identiteit <math>\cos^2 x</math>            1: <math>-\cot 2x \cdot \frac{1}{2}</math>            1: <math>+\frac{1}{2}</math>            1: <math>\sin(2x)</math></p> <p style="text-align: right;"><b>[5]</b></p>
<p>(b)</p>	$-4x^2 - 4x = 1 - (4x^2 + 4x + 1)$ $= 1 - (2x + 1)^2$ $\int \frac{dx}{\sqrt{1 - (2x + 1)^2}} = \operatorname{bgsin}(2x + 1) \cdot \frac{1}{2} + k$	<p>2: Herskryf terme            1: Faktoreer            1: <math>\operatorname{bgsin}(2x + 1)</math>            1: <math>\frac{1}{2}</math></p> <p style="text-align: right;"><b>[5]</b></p>
<p>8.2 (a)</p>	$\frac{1}{(x + 1)(x + 2)} = \frac{A}{x + 1} + \frac{B}{x + 2}$ $1 = A(x + 2) + B(x + 1)$ $x = -1: 1 = A$ $x = -2: 1 = -B$ $\frac{1}{x+1} - \frac{1}{x+2}$	<p>1: Vorm            1: kgv weg            1: A            1: B            1: Antwoord</p> <p style="text-align: right;"><b>[5]</b></p>
<p>(b)</p>	$\int_1^2 \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx = \ln(x+1) - \ln(x+2) \Big _1^2$ $= \ln \left( \frac{x+1}{x+2} \right) \Big _1^2$ $= \ln \left( \frac{3}{4} \right) - \ln \left( \frac{2}{3} \right)$ $= \ln \left( \frac{9}{8} \right) \quad A = \frac{9}{8}$	<p>2: Elke term            1: log-wet            1: Vervang            1: Antwoord            Die log-wet kon ook later toegepas word.</p> <p style="text-align: right;"><b>[5]</b></p>

<p>9.1 (a)</p>	$\Delta x_i = \frac{2}{n} \checkmark$ $x_i = \frac{2}{n} \cdot i \checkmark$ $f(x_i) = \left(\frac{2}{n} \cdot i\right)^2 - 2 \left(\frac{2}{n} \cdot i\right) \checkmark$ $\sum_{i=1}^n \left(\frac{4}{n^2} \cdot i^2 - \frac{4}{n} \cdot i\right) = \frac{4}{n^2} \sum_{i=1}^n i^2 - \frac{4}{n} \sum_{i=1}^n i \checkmark$ $= \frac{4}{n^2} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \checkmark - \frac{4}{n} \left(\frac{n^2}{2} + \frac{n}{2}\right) \checkmark$ $= -\frac{2}{3}n + \frac{2}{3n} \checkmark$ $\int_0^2 x(x-2)dx = \text{basis x hoogte}$ $= \frac{2}{n} \times \left(-\frac{2}{3}n + \frac{2}{3n}\right) = -\frac{4}{3} + \frac{4}{3n^2} \checkmark$ $\int_0^2 x(x-2)dx = \lim_{n \rightarrow \infty} \left(\frac{4}{3} + \frac{4}{3n^2}\right) = \frac{4}{3} \checkmark$ <p style="text-align: center;"><b>OF</b></p> $\Delta x_i = \frac{2}{n} \checkmark$ $x_i = \frac{2}{n} \cdot i \checkmark$ $f(x_i) = \left(\frac{2}{n} \cdot i\right)^2 - 2 \left(\frac{2}{n} \cdot i\right) \checkmark$ $\Delta x_i \times f(x_i) = \frac{4i^2}{n^3} + \frac{4i}{n^2} \checkmark$ $\sum_{i=1}^n \left(\frac{4i^2}{n^3} + \frac{4i}{n^2}\right) = \frac{4}{n^3} \sum_{i=1}^n i^2 + \frac{4}{n^2} \sum_{i=1}^n i \checkmark$ $= \frac{4}{n^3} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) \checkmark + \frac{4}{n^2} \left(\frac{n^2}{2} + \frac{n}{2}\right) \checkmark$ $= -\frac{4}{3} + \frac{4}{3n^2} \checkmark$ $\int_0^2 x(x-2)dx = \lim_{n \rightarrow \infty} \left(-\frac{4}{3} + \frac{4}{3n^2}\right) = -\frac{4}{3} \checkmark$	<p>1: <math>\Delta x_i</math>            1: <math>x_i</math>            1: <math>f(x_i)</math>            1: Sigma            1: Vervang <math>i^2</math>            1: Vervang <math>i</math>            1: Vereenvoudig            1: <math>\Delta x_i \times f(x_i)</math>            1: Antwoord</p> <p style="text-align: right;"><b>[9]</b></p>
<p>(b)</p>	<p>Oppervl = <math>\frac{4}{3} \checkmark</math></p>	<p>1: Terminologie</p> <p style="text-align: right;"><b>[1]</b></p>
<p>9.2</p>	$V = \pi \int_0^{\ln p} e^{2x} dx \checkmark = \frac{15\pi}{2}$ $\frac{e^{2x}}{2} \Big _0^{\ln p} = \frac{15}{2}$ $e^{2 \ln p} - e^0 = 15 \checkmark$ $e^{\ln p^2} - 1 = 15 \checkmark$ $p^2 = 16, \therefore p = 4 \checkmark$	<p>1: Vervang reg in formule            2: Integreer            1: Vervang waardes            1: Log-wet en <math>e^0 = 1</math>            1: Antwoord</p> <p style="text-align: right;"><b>[6]</b></p>
<p>9.3</p>	<p>Stel <math>f(x) = x - 2 - \cot\left(-\frac{x}{2}\right)</math></p> $f'(x) = 1 \checkmark - \operatorname{cosec}^2\left(-\frac{x}{2}\right) \checkmark \cdot \frac{1}{2} \checkmark$ $x_{n+1} = x_n - \frac{x_n - 2 - \cot\left(-\frac{x_n}{2}\right)}{1 - \frac{1}{2} \operatorname{cosec}^2\left(-\frac{x_n}{2}\right)} \checkmark$ $x_0 = 5,5; \quad x \approx 5,7638 \checkmark \checkmark$	<p>2: Differensieer            1: Toon Newton            1: Antwoord            1: Reg afgerond</p> <p style="text-align: right;"><b>[6]</b></p>

10.1	<p><math>x</math>-afsnit: <math>3x^2 \ln\left(\frac{x}{6}\right) = 0</math>  <math>x = 0</math> of <math>\ln\left(\frac{x}{6}\right) = 0</math> ✓                  P: <math>x = 6</math> ✓                  Gradiënt:  <math>\frac{dy}{dx} = 6x \cdot \ln\left(\frac{x}{6}\right) + 3x^2 \times \frac{1}{x}</math> ✓  <math>\left(\ln\left(\frac{x}{6}\right) = \ln x - \ln 6\right)</math> * ✓  <math>m = \frac{dy}{dx} = 6(6) \cdot \ln\left(\frac{6}{6}\right) + 3 \times 6^2 \times \frac{1}{6}</math> ✓ = <math>0 + 18</math> ✓                  OF  <math>\frac{d}{dx} \ln\left(\frac{x}{6}\right) = \frac{6}{x} \times \frac{1}{6}</math> * ✓✓</p>	<p>1: Differensieer                  1: Funksie                  1: Elke term = 0                  1: Antwoord <math>x</math>                  1: Vervang in gradiënt                  1: Antwoord</p> <p style="text-align: right;"><b>[7]</b></p>
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**Memo**

10.2 (a)	<p><math>1 - \frac{1}{1+x^2}</math> ✓  <math>= \frac{1+x^2-1}{1+x^2}</math> ✓</p>	<p>2: Elke term                  1: Antwoord</p> <p style="text-align: right;"><b>[3]</b></p>
(b)	<p>Stel <math>f = b \tan x</math> ✓ en <math>g' = x</math> ✓                  Dus <math>f' = \frac{1}{1+x^2}</math> ✓ en <math>g = \frac{x^2}{2}</math> ✓  <math>\int_0^{\sqrt{3}} x b \tan x \, dx = b \tan x \cdot \frac{x^2}{2} - \frac{1}{2} \int_0^{\sqrt{3}} \frac{x^2}{1+x^2} \, dx</math> ✓  <math>= \left( b \tan x \cdot \frac{x^2}{2} - \frac{1}{2} (x - b \tan x) \right) \Big _0^{\sqrt{3}}</math> ✓  <math>= b \tan \sqrt{3} \times \frac{3}{2} - \frac{1}{2} (\sqrt{3} - b \tan \sqrt{3}) - 0</math> ✓  <math>= \frac{\pi}{3} \times \frac{3}{2} - \frac{\sqrt{3}}{2} + \frac{\pi}{6} = \frac{2\pi}{3} - \frac{1}{2} \sqrt{3}</math></p>	<p>2: <math>f</math> en <math>g'</math>                  2: <math>f'</math> en <math>g</math></p> <p>2: Elke term                  1: Gebruik 10.2 (a)                  1: Vervang                  1: Antwoord</p> <p style="text-align: right;"><b>[9]</b></p>