

α -MATHEMATICS

Alpha Mathematics Half-year examination 2019

Grade 10

Time: 2 hours

Total: 120 marks

INSTRUCTIONS AND INFORMATION

Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 6 pages, a formula sheet of 1 page and an answer sheet of 1 page.
2. Answer ALL 8 questions.
3. Number the answers exactly the way the questions are numbered.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to two decimal places
6. Clearly show all calculations, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neat and legible.

Question 1**[20 marks]**This question must be answered **on the answer sheet**.Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 $\operatorname{Im}\left(\frac{10}{i}\right) =$

- (A) 0 (B) 10 (C) $10i$ (D) -10

1.2 If $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$ then $(g \circ f)(x) = \dots$ for all values of x .

- (A) $\sqrt{x^2} - 1$ (B) $\sqrt{x^2 - 1}$ (C) $(\sqrt{x})^2 - 1$ (D) $x - 1$

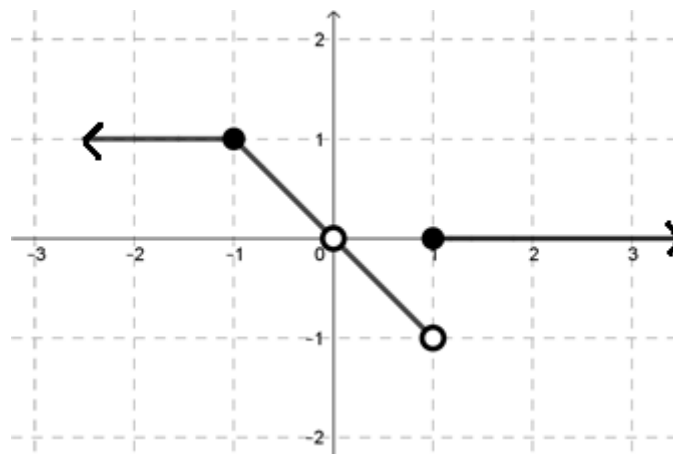
1.3 $\frac{2}{3}$ radians are equivalent to

- (A) 0.01° (B) 38.20° (C) 76.39° (D) 89.16°

1.4 The value of $i^{392} =$

- (A) 1 (B) i (C) $-i$ (D) -1

1.5 The graph of a piecewise function is given below:



The equation of the function is:

- (A) $y = \begin{cases} 1 & \text{if } x < -1 \\ -x & \text{if } -1 < x < 1; x \neq 0 \\ 0 & \text{if } x \geq 1 \end{cases}$ (B) $y = \begin{cases} 1 & \text{if } x < -1 \\ -x & \text{if } -1 \leq x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}$
- (C) $y = \begin{cases} 1 & \text{if } x \leq -1 \\ -x & \text{if } -1 < x < 1; x \neq 0 \\ 0 & \text{if } x \geq 1 \end{cases}$ (D) $y = \begin{cases} 1 & \text{if } x \leq -1 \\ -x & \text{if } -1 \leq x < 1 \\ 0 & \text{if } x \geq 1 \end{cases}$

1.6 Which one of the following statements are true:

- (A) A determinant is calculated from a matrix and the answer gives a matrix.
 (B) An identity matrix is sometimes a square matrix.
 (C) A transposed matrix is a matrix where the rows and columns exchange from the original matrix.
 (D) The numbers within a matrix are called dimensions.

1.7 What is the value of the dot product between vectors $\mathbf{a} = (3; 2)$ and $\mathbf{b} = (-3; 4)$?

- (A) 18 (B) -1 (C) -17 (D) 6

1.8 For which value of \mathbf{a} will the following matrix equation have no solution?

$$\begin{bmatrix} 2 & 3 \\ -1 & \mathbf{a} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 3 \end{bmatrix}$$

- (A) $a = -\frac{3}{2}$ (B) $a = 3$ (C) $a = -\frac{2}{3}$ (D) $a = \frac{3}{2}$

1.9 The value of a in $(a - 2i)^2 = 5 - 12i$ is,

- (A) -3 (B) ± 3 (C) -2 (D) 3

1.10 What is the horizontal component of the vector with size 10 and in the direction $\frac{\pi}{3}$ East of North.

- (A) $5\sqrt{2}$ (B) $5\sqrt{3}$ (C) 5 (D) $\frac{\sqrt{3}}{20}$

Question 2

[18 marks]

2.1 Given that $p = 5 + 7i$ and $q = 3 - 4i$. Determine the value of the following and leave your answer in the form $a + bi$ with a and b representing real numbers.

(a) $p^* - 2q$ (4)

(b) $i^{22}pq$ (4)

2.2 If $(3 - i)^2 = (x + yi)(1 + 3i)$, determine the values of x and y . (6)

2.3 Show $(-2 + 3i) + (1 - 4i)$ graphically on an axes. (4)

Question 3**[11 marks]**

Decompose $\frac{8x}{x^2(x-2)^2}$ into partial fractions. (11)

Question 4**[17 marks]**

4.1 Determine the following matrix calculations, if it is not possible, explain why.

$$(a) \begin{pmatrix} 1 & -2 \\ 4 & 0 \\ -9 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad (3)$$

$$(b) \begin{pmatrix} 3 & -1 \\ 5 & 7 \end{pmatrix} (4 \quad -6) \quad (2)$$

$$(c) 2 \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} - 5 \begin{pmatrix} 3 & -5 \\ 0 & 1 \end{pmatrix} \quad (4)$$

4.2 Determine the value(s) of m if $\begin{vmatrix} 4 & 1 & m \\ 3 & m & 1 \\ -1 & -3 & 3 \end{vmatrix} = 0$. (8)

Question 5**[11 marks]**

The following system of equations are given:

$$x - 3y = 3 \quad \text{and} \quad ax - 9y = 11$$

where a is the coefficient of x in the second equation.

5.1 Write the system in matrix form. (3)

5.2 Use Cramer's rule and show that $x = \frac{6}{3a-9}$ and $y = \frac{11-3a}{3a-9}$. Clearly show the matrices that you are using. (6)

5.3 Hence, determine the value of a if it is given that $x = 1$. (2)

Question 6**[18 marks]**

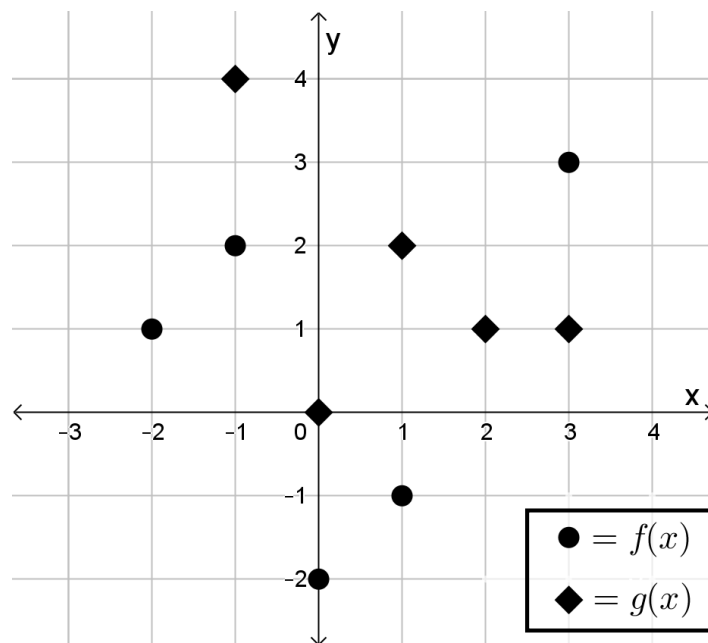
6.1 Given that $F(x) = \sqrt[4]{3-x^2} + \frac{1}{(3-x^2)^2}$.

If F is defined as $F(x) = (f \circ g)(x)$, determine the functions $f(x)$ and $g(x)$. (3)

6.2 Sketch the following piecewise function for $x \in \mathbb{R}$: (7)

$$f(x) = \begin{cases} 2 & \text{if } x < -2 \\ 1 & \text{if } x = -2 \\ -x^2 + 4 & \text{if } -2 < x < 1 \\ x + 2 & \text{if } x \geq 1 \end{cases}$$

6.3 The sketch shows the function f in \bullet and function g in \blacklozenge .



(a) Determine the value of:

(i) $f(-1)$ (1)

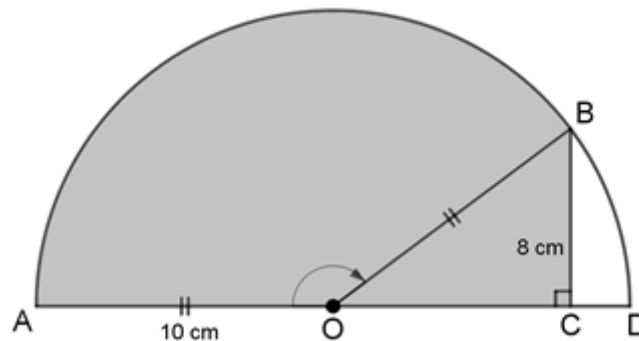
(ii) $(f \circ g)(2)$ (2)

(iii) $g(f(3))$ (2)

(b) If $F(x) = (g \circ f)(x)$ and $F(\mathbf{a}) = 2$, determine the value of \mathbf{a} . (3)

Question 7**[10 marks]**

In the diagram, ABD is a semi-circle, with midpoint O and radius 10 cm. The line BC is perpendicular on the diameter AD. $\widehat{AOB} = 2.214$ radians, $BC = 8$ cm and $AO = OB$.



- 7.1 Show that $OC = 6$ cm. (2)
- 7.2 Determine the circumference of the shaded region. (3)
- 7.3 Determine the area of the shaded region. (4)

Question 8**[15 marks]**

Given the following vectors:

$$\mathbf{u} = (3; -4)$$

$$\mathbf{w} = (-2; 1)$$

$$\mathbf{v} = (5; 6)$$

$$\mathbf{z} = (1; 4)$$

- 8.1 Determine the magnitude/size of vector \mathbf{u} . (2)
- 8.2 Determine the direction of vector \mathbf{v} with respect to the positive x -axis. (2)
- 8.3 Determine the unit vector of vector \mathbf{w} and leave your answer in root form. (3)
- 8.4 Hence, calculate the size of the angle between vectors \mathbf{u} and \mathbf{w} . (5)
- 8.5 Show that vectors \mathbf{v} and \mathbf{z} are not perpendicular to each other. (3)

- END OF THE QUESTION PAPER -

ALPHA MATHEMATICS FORMULA SHEET

MATRICES AND VECTORS:

$$\text{Cramer's rule: } x_i = \frac{|A_i|}{|A|}$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y$$

CALCULUS:

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

TRIGONOMETRY:

$$\text{In a sector: } s = r\theta \text{ and } A = \frac{1}{2}r^2\theta$$

TABLE WITH DERIVATIVES:

$F(x)$	$F'(x)$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)] \cdot g'(x)$

Alpha Mathematics Grade 10 - Half-year examination 2019**ANSWER SHEET**

Name and Surname: _____

Question Total	1 [20]	2 [18]	3 [11]	4 [17]	5 [11]	6 [18]	7 [10]	8 [15]	TOTAL 120
Learner's mark									

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D