Problem:

Evaluate the following integrals by the method of substitution:

1. $\int x^2 e^{-4x^3}\,dx$ $\quad -\frac{1}{12}e^{-4x^3}+k$

2. $\int \cos^3(x)\sin(x)\,dx$ $\quad -\frac{\cos^4 x}{4}+k$

3. $\int \frac{x^2}{7-x^3}\,dx$ $\quad -\frac{ln|7-x^3|}{3}+k$

4. $\int x^3\cos(5x^4)\,dx$ $\quad \frac{1}{20}\sin(5x^2)+k$

5. $\int \frac{\sin(x)}{2+\cos(x)}\,dx$ $\quad$ See solution

6. $\int \frac{1}{25+x^2}\,dx$ $\quad \frac{1}{5}bgtan\left(\frac{x}{5}\right)+k$

7. $\int (3x+4)^{100}\,dx$ $\quad \frac{(3x+4)^{101}}{303}+k$

8. $\int \frac{\cos(x)}{1+\sin^2(x)}\,dx$ $\quad$ See solution

9. $\int \frac{(2-\sqrt{x})^5}{\sqrt{x}}\,dx$ $\quad$ See solution

10. $\int \sec^2(2x-3)\,dx$ $\quad \frac{1}{2}\tan(2x-3)+k$

Oplossing Vraag 5

Evaluate

$$\int \frac{\sin(x)}{2+\cos(x)}\,dx$$

Let

$$u = 2 + \cos(x).$$

Then

$$du = -\sin(x)\,dx.$$

Solving for $\sin(x)dx$, we get

$$\sin(x)dx = -du$$

Hence,

$$\begin{aligned}\int \frac{\sin(x)}{2+\cos(x)}\,dx &= \int \frac{-1}{u}\,du \\ &= -\ln|u| + C \\ &= -\ln|2+\cos(x)| + C\end{aligned}$$

Oplossing Vraag 8

Evaluate

$$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx$$

Let

$$u = \sin(x).$$

Then

$$du = \cos(x)dx.$$

Hence,

$$\begin{aligned}\int \frac{\cos(x)}{1+\sin^2(x)}\,dx &= \int \frac{1}{1+u^2}\,du \\ &= \arctan(u) + C \\ &= \arctan\sin(x) + C\end{aligned}$$

Oplossing Vraag 9

Evaluate

$$\int \frac{(2-\sqrt{x})^5}{\sqrt{x}}\,dx$$

Let

$$u = 2 - \sqrt{x}.$$

Then

$$du = -\frac{1}{2\sqrt{x}}\,dx.$$

Hence,

$$\begin{aligned}\int \frac{(2-\sqrt{x})^5}{\sqrt{x}}\,dx &= \int -2u^5\,du \\ &= \frac{-2u^6}{6} + C \\ &= \frac{-(2-\sqrt{x})^6}{3} + C\end{aligned}$$